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MONETARY POLICY IN A CURRENCY UNION WITH HETEROGENEOUS COUNTRIES

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ABSTRACT

We build a two-country DSGE model for a currency union, with habit formation, product and labour differentiation and nominal rigidities. Monetary policy is defined by a rule that responds to the area's macro-variables averages weighted by each country's size. We intend to study the impact of different sources of heterogeneity between the countries (home bias in consumer preferences, wage and price mark-ups and wage and price setting rigidity) on both countries and the union. The model is calibrated and the response to shocks is simulated. The main innovation is the incorporation of several sources of heterogeneity and the assessment of its impact on welfare.

The main results of the model simulation are the following: (i) only heterogeneity regarding the home bias can lead to differentials in consumer price inflation; (ii) heterogeneity regarding wage or price mark-ups does not lead to significantly different responses to shocks of the countries; (iii) heterogeneity on nominal rigidities results in differences among the countries' response, favouring the more flexible country and resulting in smoother and longer impacts when shocks occur in the more rigid country.

We also examine the volatility of the variables and perform a formal utility-based welfare analysis. We find out that nominal rigidities are the most important source of heterogeneity. In a currency union where the central bank responds to the area wide and does not take into account national differences, it is preferable to increase flexibility in both countries and in both wages and prices, as there are significant welfare losses when countries attempt to make only wages or prices more flexible, or when only a single country is flexible. A comparison of different policy rules allows us to conclude that simpler rules (without interest rate smoothing) provide the best result in terms of welfare.

Keywords: DSGE models, currency union, monetary policy rules, heterogeneous countries, nominal rigidities, welfare

JEL Classification: E52, E58

POLÍTICA MONETÁRIA NUMA UNIÃO MONETÁRIA COM PAÍSES HETEROGÉNEOS

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RESUMO

É desenvolvido um modelo DSGE de dois países que formam uma união monetária, com hábitos no consumo, diferenciação de bens e de trabalho e rigidez nominal. A política monetária segue uma regra que responde à média das variáveis macro do agregado, ponderada pela dimensão do país. Pretende-se estudar o impacto de diferentes fontes de heterogeneidade entre os países (preferências no consumo enviesadas a favor de bens nacionais, *mark-ups* dos salários e preços e rigidez nos salários e preços) em ambos os países e na união. O modelo é calibrado e são simuladas as respostas a choques. A principal inovação consiste na incorporação de várias fontes de heterogeneidade e na avaliação do impacte em termos de bem-estar.

As simulações do modelo levam aos principais resultados: (i) apenas a heterogeneidade no enviesamento das preferências do consumo provoca diferenciais na inflação no consumidor; (ii) heterogeneidade nos *mark-ups* de preços e salários não resulta em respostas significativamente diferentes entre os países; (iii) heterogeneidade no grau de rigidez nominal implica diferentes respostas dos países, favorecendo o país mais flexível e levando a respostas mais suaves e prolongadas quando os choques ocorrem no país mais rígido.

Também se analisa a volatilidade das variáveis e o bem-estar de acordo com uma função formal derivada a partir da função utilidade. Conclui-se que a rigidez nominal é a fonte de heterogeneidade mais relevante. Numa união monetária onde o banco central responde à união e não considera as especificidades de cada país, é preferível aumentar a flexibilidade em ambos os países e nos preços e salários simultaneamente, dado que flexibilizar só salários ou preços, ou se só um país for flexível, leva a elevadas perdas de bem-estar. Conclui-se ainda que regras de política monetária simples (sem gradualismo da taxa de juro) promovem o melhor resultado de bem-estar.

Palavras-chave: modelos DSGE, união monetária, regras de política monetária, países heterogéneos, rigidez nominal, bem-estar

Classificação JEL: E52, E58

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1 Introduction

Since 1999, thirteen European countries have abandoned their national currencies and autonomous monetary policy in favour of the European Monetary Union (EMU). This can be considered as a "live experience" of the Optimal Currency Areas (OCA) theory and has motivated numerous studies on this area of research. According to this theory, it is more advantageous for various regions to share the same currency together when there is a sufficient level of synchronization and integration of trade and labour markets among the regions, i.e., when regions are similar enough. Despite the increased economic integration between European countries, namely regarding trade and financial markets, there are member countries which show persistent differentials against the euro area, namely regarding inflation and output growth. For instance, Greece, Spain and Ireland have been persistently growing above the euro area average since the mid-1990's, while Germany and Italy have remained below the average (Benalal et al., 2006). As for inflation differentials, Greece, Spain, Ireland, the Netherlands and Portugal have showed persistent positive inflation differentials while Germany, France and Austria have been situated in the opposite group (ECB, 2003). These differentials can be explained by an ongoing convergence process, but there may also exist structural differences among countries, such as different features in goods, labour or capital markets that justify diverging economic dynamics among countries.

On the other hand, the monetary policy of the euro area is defined according to the euro area as a whole. The Governing Council of the European Central Bank (ECB) has as the main objective to stabilize prices, so that inflation, measured by the Harmonized Index of Consumer Prices (HICP), remains in the medium term at a level below but close to 2%.

In this paper, we are interested in understanding the implications of having heterogeneous countries and a common monetary policy in a currency area, both for the area wide and for each member country. The model is quite general and can be applied to any currency area, although we try to approach it sometimes in the study to the euro area. Several sources of heterogeneity are considered: (i) differences in consumer preferences regarding the country where the goods are produced (home bias), since it is acceptable for consumers in the European Union to prefer to consume national goods; (ii) differences in wages and prices mark-up, as labour and product markets in euro area countries can diverge in their institutional features; and (iii) differences in wage and price setting mechanisms, as there are studies that point out the existence of different levels of nominal and real rigidity (Dhyne et al., 2005; Dickens et al., 2006, among others). In this way, our main innovation in comparison to the current literature is the incorporation of more sources of heterogeneity, particularly an interaction between wage and price rigidity, and the assessment of its impact on welfare, through the assessment of variables' volatility and through the use of a quadratic approximate welfare measure (Benigno and Woodford, 2004). This analysis is developed under the framework of a two-country Dynamic Stochastic General Equilibrium (DSGE) model, including habit formation, product and labour differentiation, monopolistic competition and nominal rigidities. This type of models is currently more frequently used for monetary policy analysis, given that these models seem to be able to replicate well the behaviour of main macroeconomic variables in response to a wide set of shocks (Smets and Wouters, 2003).

The model is calibrated and the impact of shocks[1], both common and country-specific, is simulated. We find that the source of heterogeneity that leads to the most significant differences between countries' impulse response functions is the nominal rigidity, both on wages and prices. Differences in the home bias are relevant because they are the only source of heterogeneity responsible for differences in consumer price inflation, in a context of perfectly free trade of goods among the area. Regarding the welfare analysis, we

[1] There are four shocks considered: preferences, labour supply, technology and monetary policy.

obtain interesting results. First, nominal rigidities are the source of heterogeneity which has the most important consequences on welfare. In a currency union where the central bank responds to the aggregate and does not take into account national differences, it is preferable to increase flexibility in both countries and in both wages and prices, as there are significant overall welfare losses for both economies when countries attempt to make only wages or prices more flexible, or when only a single country is flexible. Second, we find that simpler monetary policy rules seem to provide the best result in terms of welfare.

We abstract from fiscal policy issues, making the simplifying assumption that there is no government. This assumption is made since we are more interested in assessing the impacts of different sources of country heterogeneity in a currency union instead of the interactions between monetary and fiscal policy. Nonetheless, we acknowledge that when fiscal policy is taken into account, the negative impacts of asymmetries between countries or asymmetric shocks in a currency union are less relevant. Adão et al. (2006) argue that in a currency union with price rigidities, asymmetric countries or shocks and incomplete international financial markets, fiscal policy can offset the negative impacts of these aspects and lead to zero costs of a currency union, as long as labour is not mobile.

The paper is structured as follows. Section 2 provides the context on the theoretical discussions where this study can be fit in. Section 3 presents and describes the model, which will be calibrated and simulated in section 4. Under section 4, we have a first subsection (4.1) on the special case of homogeneous countries in order to try to replicate the euro area wide behaviour, while in a second subsection (4.2) the effects of heterogeneous countries are discussed. This discussion evolves in three parts: firstly, on the impacts of shocks (subsection 4.2.1); secondly, on the volatility analysis comparing various policy rules (subsection 4.2.2), and thirdly, on the consequences on welfare for each country and for the aggregate of the various sources of heterogeneity and the various policy rules considered (subsection 4.2.3). Section 5 concludes and presents directions for future research.

2 Brief summary of literature

Currently, DSGE models are frequently used for monetary policy analysis. These are mathematical models that incorporate microeconomic foundations, following a general equilibrium methodology in a dynamic and stochastic environment. This type of models evolved mainly from Real Business Cycle (RBC) models, which first appeared in the 1980s. RBC theory claimed that fluctuations in overall economic activity were due to real economic shocks, such as changes in the rate of technical progress. The RBC models were relatively small general equilibrium models, micro-founded, and were able to replicate some features regarding economic growth and fluctuations. However, the economies of these models were usually subject to a technology shock and were characterized by price flexibility and perfect competition. In this way, there was no role for monetary policy. Monetary policy only could have temporary real effects and monetary policy disturbances had no real effect since agents were able to perfectly forecast its impact in aggregate nominal expenditure. However, empirical evidence does not seem to support this result.

Empirical studies, namely through the application of Vector Autoregressive (VAR) methodology, suggest that, in the medium term, monetary policy has a temporary impact on output and a more lasting impact on inflation. Results from VAR estimation differ according to the country studied, the period chosen or the monetary policy variables used. However, two main conclusions can be drawn (Walsh, 2003):

1. Output follows a hump-shaped pattern in response to a monetary policy shock, before slowly returning to the baseline scenario and the effects of the shock have faded out. The maximum effect on output generally occurs after a relatively long lag of around 2 years.

2. A "price puzzle" arises, as an increase in the interest rate leads to an increase in the price level, at least in the short-term, decreasing slowly afterwards. A possi-

ble explanation to this puzzle is that central banks usually have more information than the rest of the agents and therefore anticipate the necessary monetary policy decisions, since they are aware of its "*long and variable lags*" before it takes effects.

In the 1990s, another type of models appeared which extended the RBC methodology and introduced nominal price and/or wage rigidity, maintaining the optimizing behaviour of agents and micro-foundations. These models are build up in a dynamic, stochastic, general equilibrium framework. This theory is also called the New-Keynesian synthesis, as it includes Keynesian features as market imperfections build up in a general equilibrium framework. Some reference papers of DSGE models with applications to monetary policy analysis are Clarida et al. (1999), McCallum and Nelson (1999), Smets and Wouters (2003). Galí (2002) overviews the main features and advantages to monetary policy analysis that DSGE models allow. These models usually share the following common features: nominal rigidity on prices and/or wages, usually with a price setting mechanism à la Calvo; monopolistic competition and product differentiation and monetary policy is usually represented by a rule for the nominal interest rate. These characteristics lead to models that can be broadly summarized in three parts: (i) the demand side is represented by an IS curve with expectations, (ii) the supply side is represent by a Phillips curve, which presents inflation with forward looking components and (iii) a policy rule for the interest rate, *ad-hoc* or optimized from the central bank's objective function.

DSGE models permit to consider a wide set of economic disturbances (Christiano et al., 2005; Smets and Wouters, 2003). Frequently, these models are estimated for a country or currency area, which make them quite useful for central bankers (Smets and Wouters, 2003). The possibility of having a structural interpretation for the movements in the data given the inclusion of various shocks, is an advantage in comparison to the VAR analysis. More recently, this type of models has also been used in forecasting, given the developments in methods which have allowed for good forecasting performance in comparison to VARs

(Smets and Wouters, 2004).

At the same time, the OCA theory was developed and "applied" in the euro area. A region is an OCA when it is advantageous to its members to share a currency together (Mundell, 1961). A group of regions will benefit from sharing a currency together if (i) shocks hitting the regions are not asymmetric, (ii) there is a high degree of labour mobility and/or wage flexibility and (iii) there is a centralized fiscal authority responsible for the redistribution of resources among the regions. When countries share a currency together, they give up their autonomous monetary policy and, therefore, one of the means to respond to shocks. If shocks hit the member countries in the same way, monetary policy response will be the right one for all countries. However, when shocks are asymmetric, the best monetary policy reaction would differ from country to country and they lack the policy adjustment mechanism. The second and third conditions work as the adjustment mechanisms of countries in a currency area when subject to asymmetric shocks: labour could move from one country to another or wages could differ between countries in order to reestablish the equilibrium; otherwise, the government could make transfers to the member countries adversely impacted by the shock.

There is a connection between economic integration and economic specialization and the costs and benefits of countries sharing the same currency. The greater the specialization of a country according to its comparative advantages, the more likely it would be hit by asymmetric shocks, which would increase the costs of an OCA. When economic integration increases, the intra-industry trade increases, decreasing the likelihood of asymmetric shocks and, therefore, increasing the benefits of an OCA as countries have economic efficiency gains from this linkage. Frankel and Rose's (1997) empirical study shows that economic integration favours business cycle synchronization, which increases the benefits of forming a currency union. Then, according to these authors, the European Union could became an OCA as it increases its degree of economic integration.

In this way, from the process of european economic integration resulted that the Treaty establishing the European Community defined the Statutes of the European System of Central Banks (ESCB), despite several authors not considering the euro area as an OCA. The primary objective of the ESCB is to attain price stability; without prejudice to this objective, the ESCB shall promote the Community policies aiming at promoting growth and employment. When the Governing Council of the ECB decides on monetary policy, it bases its decisions on developments in euro area as a whole.

Issing et al. (2001) summarizes the features of the euro area economy. Member countries share a similar industrial structure, have a significant weight of the public sector in Gross Domestic Product (GDP) and share a similar financial structure based on the banking system. On the other hand, there are significant economic imbalances among countries and regions of the euro area, with quite different levels of income per capita. Besides, countries have different institutional features regarding labour markets and the euro area as a whole is closer to the rest of the world than each individual member.

There are empirical studies showing that euro area countries are not homogenous. According to Benalal et al. (2006), the GDP growth dispersion among euro area countries remains fairly stable, with some countries systematically above or below the euro area's average. These differences seem to reflect structural differences among countries, as the business cycle synchronization has increased, which favours Frankel and Rose's (1997) thesis. Inflation also differs among countries, as we observe persistent inflation differentials in the euro area. These inflation differentials are due to some non-structural factors (differences in profit margin and unit labour cost developments, in administered price and indirect tax changes and in cyclical positions), but also to structural factors, such as the impact of nominal convergence effects triggered by EMU, structural rigidities and, to a limited extent, income convergence and Balassa-Samuelson effects (ECB, 2003).

Markets structure and functioning also differs among member countries. For instance, price and wage setting, which is one of the sources of heterogeneity considered in our model, seems to be different across euro area countries. Dhyne et al. (2005) find that heterogeneity in price setting behaviour across countries is relevant, although partially related to differences in the consumption structure. This also suggests that there is also heterogeneity regarding consumer preferences, another source of heterogeneity considered in our model. Differences among countries seem to be more significant and pronounced regarding wage rigidity. Dickens et al. (2006) find that there are substantial differences across countries[2], namely in the degree of downwards real and nominal wage rigidity.

This type of cross-country asymmetries has implications on the effects of monetary policy. Recently, literature has been studying the effects of heterogeneity among the regions of a currency union using DSGE models for more than one country. Usually, two-country models are developed, with the usual features: product differentiation, monopolistic competition, nominal rigidities and complete financial markets. The countries in the models are open to each other while closed to the rest of the world. Monetary policy also follows what happens in the euro area, with a common central bank setting the interest rate according to a rule which usually is a function of the currency area's average inflation and output. Then, some distinct aspects between countries are introduced.

Benigno (2004) builds a two-region model with product differentiation between regions, monopolistic competition, price rigidity and labour immobility between regions. Regions are subject to asymmetric shocks. He also defines a welfare criterion in order to evaluate monetary policy in the currency area. It is found that the optimal monetary policy should lead to a high degree of inflation inertia. When the monetary policy authority responds to the average of the area, weighted by the regions size, the optimal is not attained. But the optimal plan can be approximated by an inflation targeting policy which gives a higher weight to the inflation in the region where nominal rigidities are more striking.

[2] They study 15 European countries, of which 9 belong to the euro area, and the USA.

Gomes (2004) studies the implications for a currency area of different price rigidity degrees in response to common and specific shocks by using a calibrated model. The degree of price rigidity differs between countries. Shocks, either common or specific, lead to significant differentials between countries, which are larger when shocks are idiosyncratic. These differentials are favourable to the less sluggish country. When the more rigid country is hit by the shock, impacts are smoother, mainly in the other country. She also compares different monetary policy rules. Rules that result in the best outcome for aggregate variables do not mean that they also lead to the best individual result. Interest rate smoothing stabilizes inflation and output, reduces countries differentials, but it also reduces inflation correlation between countries. A rule which responds to the output gap diminishes output volatility with prejudice for inflation volatility and decreases output volatility in the more rigid country while increasing it in the other, reducing, therefore, output correlation between the countries.

Other contributions focus more on labour market specificities. Abbritti (2007) studies how countries in a currency union with different institutional features regarding labour markets respond to shocks, either common or specific, in a calibrated model. In his model, there is also rigidity in real wages (Hall's wage norm) and there is no migration between countries. The real wage rigidity increases the persistence of temporary shocks and contributes to explain the lasting inflation and unemployment differentials. Additionally, more sclerotic labour markets increase inflation volatility and reduce unemployment volatility. The author also finds that common (monetary policy) shocks can also have large asymmetric effects. Finally, strict inflation targeting is found not to be optimal, as it leads to large and persistent unemployment fluctuations, either at the aggregate or individual levels.

There is another line of investigation more focused on estimation of these type of models for the euro area, for example Jondeau and Sahuc (2006) and Pytlarczyk (2005).

The first of these papers develops a model with habit formation, price rigidity à la Calvo, product differentiation and home bias in consumption goods. It also allows for different preferences and technologies between countries, although with labour as the only input in production. They then estimate the model for some European countries (Germany, France and Italy) and analyze the costs, in terms of welfare, of ignoring the member countries' heterogeneity. They find out that in the case of an optimal monetary policy based on the area wide model, then there are relatively larger welfare losses than when the optimal rule is derived from the multi-country model. Then, the central bank should take into consideration the specific features of the countries forming the currency area, both in terms of behavioral parameters and specific shocks, as the welfare losses are due to the use of a sub-optimal forecasting model, instead of the use of a policy rule which responds to the aggregate variables. Additionally, they also find out that introducing interest rate smoothing in the welfare function does not change significantly the results.

Pytlarczyk's (2005) estimated model for Germany and the remaining euro area seems to allow a consistent analysis of the interaction between these two regions. This model is slightly more evolved than Jondeau and Sahuc (2006), as it includes some features present in Smets and Wouters (2003) such as capital and an adjustment cost of capital, price and wage rigidity à la Calvo, product and labour differentiation and external habit formation.

The model presented in our paper follows the line of research of the above mentioned papers, and goes beyond them by mixing some features as wage and price rigidity and home bias in consumer preferences with the purpose of comparing different ad-hoc monetary policy rules in terms of their impact in each country and in the main aggregate variables and welfare. In this way, the main innovation of this paper refers to the combined analysis of price and wage rigidity in heterogeneous countries, showing the impacts on welfare from changing the flexibility of price and wage setting mechanisms. We also consider the consequences of having different preferences in consumption and different efficiency degrees in labour and goods markets (through differences in wage and price mark-ups), although these sources of heterogeneity have a smaller impact on welfare. Finally, we consider the impact of different monetary policy rules, all of which respond to the aggregate average variables, as it occurs in the euro area.

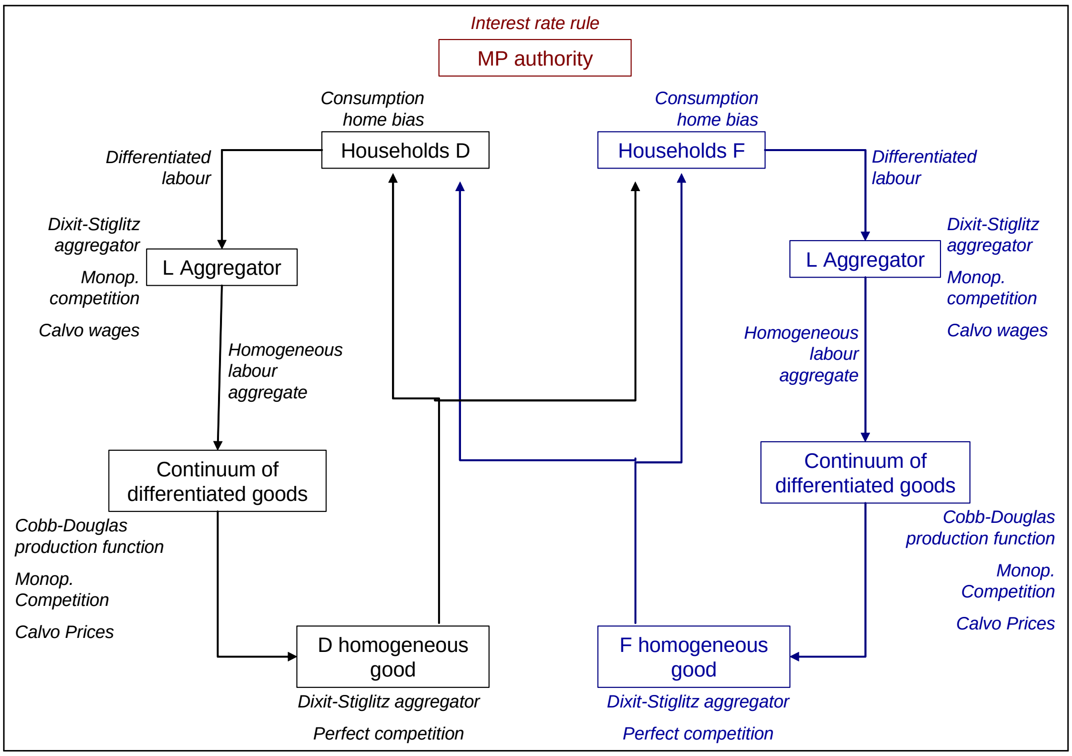
3 Description of the model

We build a currency union model consisting of two economies, the domestic economy (variables denoted by D and parameters without an asterisk) and the foreign economy (variables denoted by F and parameters with an asterisk). The two countries form together a currency area, meaning that they share the same currency and have a common central bank that implements monetary policy for the aggregate. The population of the aggregate area is a continuum of identical and infinitely lived agents in the interval $[0, 1]$ (aggregate size is then normalized to 1), which produce a bundle of differentiated goods. Households are denoted by j. When these live in the domestic economy, we have $j \in [0, n]$, while when they live in the foreign economy $j \in (n, 1]$. Therefore, n is the relative size of the home economy. Similarly, each firm produces one differentiated good i. Then, index i denotes both firms and goods. Relative output size of each economy corresponds to the relative population size. When $i \in [0, n]$, the firm belongs to the home economy and when $i \in (n, 1]$, it belongs to the foreign economy. In order to easily identify where the household or firm belong to, whenever considered adequate, we will denote the household or firm belonging to the foreign country with an asterisk (j^* or i^*).

All goods are tradable, there is free trade of goods and financial markets are complete. However, labour markets are specific to each country, i.e., there is no mobility of labour between the two countries. Figure 1 presents a schematic representation of the model.

The model follows closely DSGE models used recently in the literature. Indeed, we follow closely the structure of Smets and Wouters (2003) for the euro area. However,

Figure 1: Schematic representation of the model



we simplify by excluding capital from our model. Adjustment between countries occurs through the goods market, since labour is restricted to each country. The Smets and Wouters (2003) model structure is applied to a two-country model, which brings it close to Benigno (2004) and Jondeau and Sahuc (2006). These models assume optimal wage setting, while in the model we develop here we introduce rigidity in wage setting. Evidence from the euro area suggests that wages are rigid, specially downwards, and that wage rigidity differs between countries (Dickens et al., 2006). Both prices and wages are set according to a Calvo mechanism with indexation to past inflation. Calvo price/wage setting mechanism is the most commonly mean of introducing price/wage rigidity in the recent literature. It is an useful feature as it can be solved without explicitly tracking the distribution of prices across firms and it is able to capture the factors that contribute to nominal stickiness (Christiano et al., 2005). Other models of staggered price setting introduce costs of price adjustment (Rotemberg, 1982, 1996). Although the Rotemberg sticky price model and the Calvo price model lead to a similar Phillips curve, they have different implications for micro data: the Rotemberg model implies a single price in the micro data, while the Calvo model has implicit a distribution of prices among firms, which seems to be more plausible. There are also models with staggered price setting using Taylor contracts, which imply a certain time between price adjustments (Chari et al., 2000), but can be criticized by not accounting for the sluggish response of inflation adjustment. Regarding wage stickiness, we find in the literature the introduction of Taylor wage contracts, which will impact on price rigidity. Indeed, with these wage contracts, prices display inertia but the inflation rate does not need to show inertia (Walsh, 2003).

3.1 Households and consumption

Households in each economy consume differentiated goods, either produced internally or imported. All households within each economy share the same preferences and endowments but supply to firms resident in the respective country differentiated labour. There is heterogeneity in households behaviour across countries. The representative household j in both economies seeks to maximize its expected discounted utility, where the discount factor β is common to the area wide:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t(j) \tag{1}$$

The instantaneous utility function $U_t(j)$ shares the same functional form for both economies, depending positively on consumption $\left(C_t^D, C_t^F\right)$ and negatively on labour $\left(L_t^D, L_t^F\right)$; but it can differ in respect to the consumption habit $\left(H_t^D, H_t^F\right)$, the relative risk aversion coefficients on consumption (σ_c, σ_c^*) and on labour supply (σ_l, σ_l^*) and the preference $\left(\varepsilon_t^{bD}, \varepsilon_t^{bF}\right)$ and labour supply $\left(\varepsilon_t^{LD}, \varepsilon_t^{LF}\right)$ shocks.

The instantaneous utility function of the representative household j in the domestic economy and j^* in the foreign one are, respectively:

$$\begin{aligned} U_t^D(j) &= \varepsilon_t^{bD}\left[\frac{1}{1-\sigma_c}\left(C_t^D(j)-H_t^D\right)^{1-\sigma_c}-\frac{\varepsilon_t^{LD}}{1+\sigma_l}\,L_t^D(j)^{1+\sigma_l}\right] \\ U_t^F(j^*) &= \varepsilon_t^{bF}\left[\frac{1}{1-\sigma_c^*}\left(C_t^F(j^*)-H_t^F\right)^{1-\sigma_c^*}-\frac{\varepsilon_t^{LF}}{1+\sigma_l^*}\,L_t^F(j^*)^{1+\sigma_l^*}\right] \end{aligned} \tag{2}$$

Households present external habit formation, in the sense that their utility depends not only on current consumption but also on how it differs from a proportion of the previous period total consumption in the respective economy: $H_t^D = hC_{t-1}^D$ and $H_t^F = h^*C_{t-1}^F$, where h (h^*) is the habit persistence parameter (in line with the "catching up with the Joneses" argument presented in Abel, 1990). Following the recent literature (Smets and Wouters, 2003; among others), including external habit persistence seems to provide a greater correspondence to consumption behaviour, which seems to be more persistent and to respond more gradually to economic shocks, as consumers dislike large and rapid changes in consumption or can not change their consumption rapidly (especially if we consider goods such as housing) (Fuhrer, 2000).

Consumers in both countries consume goods produced in either country. Thus, total consumption in the home economy includes the consumption of internally produced goods $(C_{D,t})$ and imported goods $(C_{F,t})$ from the foreign country. The consumption baskets of the representative households in the home and foreign economies are given by (following Jondeau and Sahuc, 2006):

$$C_t^D(j) = \frac{\left(C_{D,t}(j)\right)^{\varpi}\left(C_{F,t}(j)\right)^{1-\varpi}}{\varpi^{\varpi}\left(1-\varpi\right)^{(1-\varpi)}} \tag{3}$$

$$C_t^F(j^*) = \frac{\left(C_{D,t}(j^*)\right)^{\varpi^*}\left(C_{F,t}(j^*)\right)^{1-\varpi^*}}{\varpi^{*\,\varpi^*}\left(1-\varpi^*\right)^{(1-\varpi^*)}} \tag{4}$$

In our model we include the hypothesis of the existence of home bias in preferences, given by the parameter ω (ω^*). In this way, ω (ω^*) is the share of domestically produced goods in the total consumption in the home economy (remaining euro area). In case

$\omega = 0.5$ ($\omega^* = 0.5$), then consumers do not distinguish goods by the country were they were produced. In case $\omega > 0.5$ ($\omega^* > 0.5$), consumers prefer goods produced in the home economy ($C_{D,t}$), i.e., we have a home bias in consumption preferences. In case $\omega < 0.5$ ($\omega^* < 0.5$), consumers prefer goods produced in the foreign economy ($C_{F,t}$).

$C_{D,t}(j)$ ($C_{F,t}(j)$) is the consumption by household j of domestically (foreign) produced goods, which are imperfect substitutes and can either be consumed in the country where they are produced in or can be exported and consumed by households in the other country. In this way, $C_{D,t}(j)$ ($C_{F,t}(j)$) is an index given by the following CES aggregator:

$$\begin{aligned} C_{D,t}(j) &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_{D,t}(i,j)^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}} \\ C_{F,t}(j) &= \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta^*}} \int_n^1 C_{F,t}(i^*,j)^{\frac{\theta^*-1}{\theta^*}}\, di^*\right]^{\frac{\theta^*}{\theta^*-1}} \end{aligned} \tag{5}$$

$C_{D,t}(i,j)$ ($C_{F,t}(i^*,j)$) is the consumption by household j of the generic good i (i^*) produced in the home (foreign) economy. Since goods produced in the same country are differentiated, θ (θ^*) denotes the elasticity of substitution between goods produced in home (foreign) economy. These goods are then aggregated taking into account the product differentiation and the production size of the country where they are produced.[3]

Households maximize their intertemporal utility function subject to an intertemporal budget constraint which states that their income from labour, dividends and financial markets applications will be fully used every period for consumption and transactions in financial markets. The total income of each household is given by the labour income, where $w_{D,t}$ ($w_{F,t}$) denotes the real wage, and dividends from participating in the imperfectly

[3] Take also note of the following definitions, which will be useful for the market equilibrium conditions:

$C_{D,t}^D = \int_0^n C_{D,t}(j)\, dj$ as the total consumption of home produced goods in the home economy;

$C_{D,t}^F = \int_n^1 C_{D,t}(j^*)\, dj^*$ as the total consumption of home produced goods in the foreign economy;

$C_{F,t}^D = \int_0^n C_{F,t}(j)\, dj$ as the total consumption of foreign produced goods in the home economy;

$C_{F,t}^F = \int_n^1 C_{F,t}(j^*)\, dj^*$ as the total consumption of foreign produced goods in the foreign economy.

competitive firms (Div_t^D for households in D and Div_t^F for households in F), which are denominated in units of the consumption basket of the respective country. Households also receive income from the applications made in the bond market in the previous period. Indeed, households can trade area wide riskless bonds B_t. These are one-period bonds with price b_t. Each domestic (foreign) household detains the same amount of $B_t(j)$ $(B_t(j^*))$. The bond market must be balanced in order to avoid the existence of Ponzi games. Therefore, the financial market is at equilibrium at each moment when the area aggregate is neither at a debtor nor at a creditor position $(nB_t(j) + (1-n) B_t(j^*) = 0)$. The transversality condition holds and neither of the economies individually can assume a permanent debtor or creditor position ($\lim\limits_{t\to\infty} b_t n B_t(j) = 0$ and $\lim\limits_{t\to\infty} b_t (1-n) B_t(j^*) = 0$). The income from applications in bonds made in the previous period and the transactions made in the current period are defined in nominal terms. In order to make it consistent with the remaining budget constraint, we have to divided it by P_t^{cD} (P_t^{cF}), the consumer price index for the home (foreign) economy.

Households also have access to a country-specific state-contingent security S_t^D (S_t^F), denominated in units of the domestic (foreign) consumption basket. This security allows an harmonization between households in the same country, so that all the households are similar regarding consumption and area wide common asset holdings. Within each region, there is a zero net supply of these state-contingent securities, i.e., $\int_0^n S_t^D(j)\,dj = 0$ and $\int_n^1 S_t^F(j^*)\,dj^* = 0$. In this way, we define the intertemporal budget constraint as follows (for domestic households and for foreign households, respectively):

$$\begin{aligned} b_t \frac{B_t(j)}{P_t^{cD}} &= \frac{B_{t-1}(j)}{P_t^{cD}} + w_{D,t}(j) L_t^D(j) + Div_t^D(j) + S_t^D(j) - C_t^D(j) \qquad (6) \\ b_t \frac{B_t(j^*)}{P_t^{cF}} &= \frac{B_{t-1}(j^*)}{P_t^{cF}} + w_{F,t}(j^*) L_t^F(j^*) + Div_t^F(j^*) + S_t^F(j^*) - C_t^F(j^*) \end{aligned}$$

The optimization problem of the representative household in each economy resumes to the maximization of equation (1) with respect to consumption and bond holdings, subject to the constraint (6), i.e., corresponds to the optimization of the following Lagrangian:

$$\pounds = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \varepsilon_t^{bD} \left[\frac{1}{1-\sigma_c} \left(C_t^D (j) - H_t^D \right)^{1-\sigma_c} - \frac{\varepsilon_t^{LD}}{1+\sigma_l} L_t^D (j)^{1+\sigma_l} \right] - \\ -\lambda_{D,t} \left(\frac{b_t B_t (j) - B_{t-1} (j)}{P_t^{cD}} - w_{D,t} (j) L_t^D (j) - Div_t^D (j) - S_t^D (j) + C_t^D (j) \right) \end{array} \right\}$$

The first-order conditions yield the usual Euler equation, i.e., the optimal consumption flows over time, when we derive the Lagrangian with respect to B_t (equation 7), and the marginal utility from consumption when we derive the Lagrangian with respect to C_t^D (equation 8), where $R_t = \frac{1}{b_t}$ is the gross nominal rate of return on bonds.

$$E_t \left[\beta \frac{\lambda_{D,t+1}}{\lambda_{D,t}} \frac{R_t P_t^{cD}}{P_{t+1}^{cD}} \right] = 1 \tag{7}$$

$$\lambda_{D,t} = \varepsilon_t^{bD} \left(C_t^D - H_t^D \right)^{-\sigma_c} \tag{8}$$

Similar equations are found for the foreign economy.

3.2 Labour supply and wages

Households provide labour services to the firms and the type of labour that each household provides is different from each other, so that households are competitive monopolist suppliers of differentiated labour. In other words, labour services provided by one household are assumed by firms to be different from the labour services provided by another household from the same country (Erceg et al., 2000). In this way, households sell their labour services to firms and set their wages in each period in order to maximize their utility given their budget constraint.

Labour markets are specific to each economy and there is no mobility of labour between the home and the foreign economy. The way labour markets function in the two economies is similar, although with different degrees of reaction (different parameters but with the same functional form). There is rigidity on wages and in both economies wages are set following a Calvo mechanism. This means that not all households can adjust their wages optimally on every period. A particular household can reoptimize its nominal wage only when it receives a signal to do so. When optimizing the new wage, the household takes into account the likelihood of being able to reoptimize its wage again in the future (Erceg et al., 2000; Smets and Wouters, 2003). When the household does not receive this signal to reoptimize, it adjusts wages as a function of past inflation.

The probability of a household in the home economy receiving the signal to optimize its nominal wage to $\tilde{W}_{D,t}(j)$ is given by $1-\xi_w^D$. The rest of the households are not able to reoptimize their wages (with probability ξ_w^D) and therefore can only adjust their wages according to past inflation. Then, the wage is set as follows:

$$W_{D,t}(j) = \begin{cases} \tilde{W}_{D,t}(j) \text{ with probability } 1-\xi_w^D \\ \left(\frac{P_{t-1}^{cD}}{P_{t-2}^{cD}}\right)^{\gamma_w} W_{D,t-1}(j) \text{ with probability } \xi_w^D \end{cases} \tag{9}$$

The parameter γ_w in equation (9) is the degree of wage indexation. When $\gamma_w = 0$ there is no indexation and the wages that can not be reoptimized remain constant; if $\gamma_w = 1$, it implies perfect indexation to past consumer inflation.

Given labour differentiation, domestic firms transform the differentiated labour services provided by domestic households into a labour index L_t^D that follows a Dixit-Stiglitz type aggregator (Erceg et al., 2000):

$$L_t^D = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varphi}} \int_0^n \left(L_t^D(j)\right)^{\frac{\varphi-1}{\varphi}} dj\right]^{\frac{\varphi}{\varphi-1}} \tag{10}$$

$L_t^D(j)$ is the labour supplied by household j. The parameter φ represents the elasticity of substitution between different types of labour services within the home economy. The nominal wage index at home ($W_{D,t}$) is also defined in a similar way:

$$W_{D,t} = \left[\frac{1}{n}\int_0^n \left(W_{D,t}(j)\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \tag{11}$$

$W_{D,t}(j)$ denotes the nominal wage negotiated by household j at date t. In this way, household j faces the following labour specific demand from all firms in the home economy:

$$L_t^D(j) = \left(\frac{W_{D,t}(j)}{W_{D,t}}\right)^{-\varphi} \frac{L_t^D}{n} \tag{12}$$

Households set their wages in order to optimize their intertemporal objective function (1) subject to the budget constraint (6) and the above labor demand (12). The optimization problem leads to the following mark-up equation for the optimized nominal wage $\tilde{W}_{D,t}$[4]:

$$\frac{\tilde{W}_{D,t}}{P_t^{cD}} E_t \sum_{k=0}^{\infty} \left(\beta \xi_w^D\right)^k \frac{\varphi - 1}{\varphi} \tilde{L}_{t+k}^D U'_{C_{t+k}^D} \frac{\left(\dfrac{P_{t+k-1}^{cD}}{P_{t-1}^{cD}}\right)^{\gamma_w}}{\dfrac{P_{t+k}^{cD}}{P_t^{cD}}} = -E_t \sum_{k=0}^{\infty} \left(\beta \xi_w^D\right)^k \tilde{L}_{t+k}^D U'_{L_{t+k}^D} \tag{13}$$

Equation (13) states that the optimal wage will be set at the level where the disutility from an additional working hour equals a mark-up over the increase in marginal utility from consumption due to the higher working hours. The terms $U'_{C_{t+k}^D}$ and $U'_{L_{t+k}^D}$ denote, respectively, the marginal utility from consumption and the marginal disutility from labour[5] and $\tilde{L}_{t+k}^D$ is the labour supplied at moment $t+k$ by household j which reoptimized its wage at moment t, i.e., is the labour supply in time given that the household was allowed to reoptimize its wage in the initial moment.

The law of motion of the home aggregate index of the nominal wage can be derived from equations (9) and (11) is given by:

[4] Similarly to what happens in the price setting, the average optimal wage equation will be similar to the individual optimal wage. Although households provide differentiated labour and, therefore, face different wages, they still share the same utility function and the same budget constraint and are not subject to individual shocks. Thus, we can consider them as representative households in the sense that the decision they take is the same and they will all choose the same wage (Erceg et. al, 2000; Woodford, 2003).

[5] Recall that $U'_{C_{t+k}^D} = \varepsilon_{t+k}^{bD}(C_{t+k}^D(j) - H_{t+k}^D)^{-\sigma_c}$ and $U'_{L_{t+k}^D} = \varepsilon_{t+k}^{bD}\varepsilon_{t+k}^{LD}\left(L_{t+k}^D\right)^{\sigma_L}$.

$$(W_{D,t})^{1-\varphi} = \xi_w^D \left[W_{D,t-1} \left(\frac{P_{t-1}^{cD}}{P_{t-2}^{cD}} \right)^{\gamma_w} \right]^{1-\varphi} + \left(1-\xi_w^D\right) \left(\tilde{W}_{D,t}\right)^{1-\varphi} \tag{14}$$

We get similar expressions for the foreign economy. The wage indexation for the households which can not reoptimize is $W_{F,t}(j^*) = \left(\frac{P_{t-1}^{cF}}{P_{t-2}^{cF}} \right)^{\gamma_w^*} W_{F,t-1}(j^*)$. The labour demand in this country is given by $L_t^F(j^*) = \left(\frac{W_{F,t}(j^*)}{W_{F,t}} \right)^{-\varphi^*} \frac{L_t^F}{1-n}$, while aggregate labour and aggregate wage are defined by the equations:

$$\begin{aligned}
L_t^F &= \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varphi^*}} \int_n^1 \left(L_t^F(j^*) \right)^{\frac{\varphi^*-1}{\varphi^*}} dj^* \right]^{\frac{\varphi^*}{\varphi^*-1}} \\
W_{F,t} &= \left[\frac{1}{1-n} \int_n^1 \left(W_{F,t}(j^*) \right)^{1-\varphi^*} dj^* \right]^{\frac{1}{1-\varphi^*}}
\end{aligned}$$

Taking the optimization problem similar to the home economy, i.e., the maximization of the intertemporal utility function regarding wage subject to the budget constraint and the labour demand equation, we reach a similar mark-up equation for the optimized wage in the foreign country:

$$\frac{\tilde{W}_{F,t}}{P_{F,t}} E_t \sum_{k=0}^{\infty} \left(\beta \xi_w^F\right)^k \frac{\varphi^*-1}{\varphi^*} \tilde{L}_{t+k}^F U'_{C_{t+k}^F} \frac{\left(\frac{P_{t+k-1}^{cF}}{P_{t-1}^{cF}} \right)^{\gamma_w^*}}{\frac{P_{t+k}^{cF}}{P_t^{cF}}} = -E_t \sum_{j=0}^{\infty} \left(\beta \xi_w^F\right)^k \tilde{L}_{t+k}^F U'_{L_{t+k}^F} \tag{15}$$

The law of motion of the foreign aggregate index of the nominal wage follows

$$(W_{F,t})^{1-\varphi^*} = \xi_w^F \left[W_{F,t-1} \left(\frac{P_{t-1}^{cF}}{P_{t-2}^{cF}} \right)^{\gamma_w^*} \right]^{1-\varphi^*} + \left(1-\xi_w^F\right) \left(\tilde{W}_{F,t}\right)^{1-\varphi^*} \tag{16}$$

3.3 Firms

There is a continuum of imperfectly competitive firms. Firms producing in the home economy belong to the interval $[0, n]$, while firms producing in the foreign country belong to the interval $(n, 1]$. Similarly to what was defined for households, firms show different behaviour according to the economy they belong to. This means that equations summarizing firms' behaviour share the same functional form but with different parameters. Firms produce differentiated goods which are bundled into homogeneous domestic and foreign baskets of goods which can be freely traded among the area wide. The homogeneous domestic and foreign goods (Y_t^D and Y_t^F) are given by the following Dixit-Stiglitz type aggregator, which takes into account households preferences (Erceg et al., 2000):

$$Y_t^D = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n Y_t^D(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \text{ and } Y_t^F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta^*}} \int_n^1 Y_t^F(i^*)^{\frac{\theta^*-1}{\theta^*}} di^*\right]^{\frac{\theta^*}{\theta^*-1}} \tag{17}$$

The domestic firm i only produces one good i, which is differentiated from the goods produced by other domestic firms. Firms only have one productive factor, labour. In this way, and for the home economy, the individual production function ($Y_t^D(i)$) depends on labour ($L_t^D(i)$), productivity (A_t^D) and fixed costs (Φ^D).[6]

$$Y_t^D(i) = A_t^D L_t^D(i) - \frac{\Phi^D}{n} \tag{18}$$

Productivity differs from economy to economy, but in both it is assumed to follow an AR(1) process: $A_t^D = (1-\rho_a)\bar{A}^D + \rho_a A_{t-1}^D + \eta_{a,t}$, where $\bar{A}^D$ is the long-term productivity, ρ_a is the persistence parameter and $\eta_{a,t}$ is a random shock.

Cost minimization conditions imply that the demand from all households for goods produced by firm i follows:

$$Y_t^D(i) = \left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\theta} \frac{Y_t^D}{n} \tag{19}$$

where $P_{D,t}(i)$ is the price of good i and $P_{D,t}$ is the price index of home produced

[6] Following Christiano et al. (2005), fixed costs are included so that there is a minimum level of production that covers the fixed costs. Also in Christiano et al. (2005), it is argued that fixed costs are introduced so that steady-state profits are null.

goods. All firms share the same marginal costs, given by the nominal wage weighted by productivity:

$$MC_t^D = \frac{W_{D,t}}{A_t^D} \tag{20}$$

Similarly to what was defined for wages, prices are also set by firms according to a Calvo mechanism. At each period t, firm i is allowed to reoptimize and chooses the price $\tilde{P}_{D,t}(i)$ with probability $\left(1-\xi_p^D\right)$. If the firm does not receive the signal to reoptimize (with probability ξ_p^D), then it sets its price according to past inflation. In this way, price is set according to

$$P_{D,t}(i) = \begin{cases} \tilde{P}_{D,t}(i) \text{ with probability } 1-\xi_p^D \\ \left(\frac{P_{D,t-1}}{P_{D,t-2}}\right)^{\gamma_p} P_{D,t-1}(i) \text{ with probability } \xi_p^D \end{cases} \tag{21}$$

The low branch of equation (21) gives the expression for the price adjustment indexed to previous period producer price inflation for domestic firms which can not reoptimize their wages at period t. The parameter γ_p is the degree of price indexation to past inflation. If $\gamma_p = 0$, then no adjustment is made and prices remain fixed; on the other hand, when $\gamma_p = 1$, prices adjust perfectly to the previous period observed inflation.

The price index of goods produced by all domestic firms is given by the following Dixit-Stiglitz type aggregator:

$$P_{D,t} = \left[\frac{1}{n}\int_0^n P_{D,t}(i)^{1-\theta}\, di\right]^{\frac{1}{1-\theta}} \tag{22}$$

The firm's objective is to set the price which maximizes its profits, taking into account the probability it has to reoptimize its price in the future. This will lead to a mark-up equation for the optimal price $\tilde{P}_{D,t}(i)$ for each firm. Since this is a representative agent model, then every agent will follow the same "rule" for the price formation as there are no individual specific shocks (Woodford, 2003; Jondeau and Sahuc, 2006; Pytlarczyk, 2005), and the individual price mark-up is defined in aggregated terms for the home economy as follows:

$$E_t \sum_{k=0}^{\infty} \left(\beta \xi_p^D\right)^k \rho_{t+k} Y_{t+k}^D \left[\frac{\tilde{P}_{D,t}}{P_{D,t}} \frac{P_{D,t}}{P_{D,t+k}} \left(\frac{P_{D,t+k-1}}{P_{D,t-1}} \right)^{\gamma_p} - \frac{\theta}{\theta - 1} mc_{t+k}^D \right] = 0 \tag{23}$$

where $\rho_{t+k} = \dfrac{U'_{C_{t+k}^D}}{U'_{C_t^D}}$ and $\beta \rho_t$ is the discount factor used by the shareholders-households and mc_t^D are the real marginal costs. Equation (23) states that the optimal price will be set at a level consistent with the mark-up over current and expected future marginal costs.

Taking into account that prices are set according to a Calvo mechanism, then we can define the law of motion of the home producer price index, derived from equations (21) and (22):

$$(P_{D,t})^{(1-\theta)} = \xi_p^D \left[P_{D,t-1} \left(\frac{P_{D,t-1}}{P_{D,t-2}} \right)^{\gamma_p} \right]^{(1-\theta)} + \left(1 - \xi_p^D\right) \left(\tilde{P}_{D,t}\right)^{(1-\theta)} \tag{24}$$

The way foreign firms set their prices is quite similar to home firms, with the caveat of the different parameters. Therefore, the foreign mark-up equation for the optimal price $\tilde{P}_{F,t}$ is given by:

$$E_t \sum_{k=0}^{\infty} \left(\beta \xi_p^F\right)^k \rho_{t+k}^* Y_{t+k}^F \left[\frac{\tilde{P}_{F,t}}{P_{F,t}} \frac{P_{F,t}}{P_{F,t+k}} \left(\frac{P_{F,t+k-1}}{P_{F,t-1}} \right)^{\gamma_p^*} - \frac{\theta^*}{\theta^* - 1} mc_{t+k}^F \right] = 0 \tag{25}$$

and the foreign producer price index follows a similar law of motion:

$$(P_{F,t})^{(1-\theta^*)} = \xi_p^F \left[P_{F,t-1} \left(\frac{P_{F,t-1}}{P_{F,t-2}} \right)^{\gamma_p^*} \right]^{(1-\theta^*)} + \left(1 - \xi_p^F\right) \left(\tilde{P}_{F,t}\right)^{(1-\theta^*)} \tag{26}$$

The law of one price holds for each good individually taken, meaning that it will have the same price independently where it is consumed. Then, producer prices are the prices at which goods are sold, either at home or in the foreign country. For example, firm i sets the price of good i, which is produced domestically, and sells this good at price $P_{D,t}\left(i\right)$, which is the same price for domestic consumers and for foreign consumers. Consumer price indexes are determined according to the share of home produced goods and imported goods in the consumption basket (ϖ):

$$P_t^{cD} = \left(P_{D,t}\right)^{\varpi} \left(P_{F,t}\right)^{(1-\varpi)} \text{ and } P_t^{cF} = \left(P_{D,t}\right)^{\varpi^*} \left(P_{F,t}\right)^{(1-\varpi^*)} \tag{27}$$

Given equations (27) for each economy consumer price indexes and the law of one price, then we can establish a relation between consumer and producer prices in both economies. Consumer price index can differ between countries due to the existence of a home bias in consumption, i.e., PPP will not hold whenever $\varpi \neq \varpi^*$[7]. This differs from Benigno (2004), which assumes that PPP holds since he does not consider the existence of home bias and the share of each countries goods in the consumption basket equals the countries' size.

$$\frac{P_t^{cF}}{P_t^{cD}} = \left(\frac{P_{F,t}}{P_{D,t}}\right)^{\varpi-\varpi^*} \tag{28}$$

The model does not consider the hypothesis of pricing-to-market (Obstfeld and Rogoff, 1996; Betts and Devereux, 2000), as price discrimination is not feasible given the characteristics of the currency union. The two countries belong to the same currency union with free trade in the goods market. We do not consider transaction costs or trade tariffs and there is no nominal exchange rate as countries share the same currency. If firms would decide to set prices for the exported goods differently from the goods sold domestically, then there would be arbitrage opportunities and the prices of goods exported would tend to approach the price of the goods consumed internally. There can be indeed differences between the prices of the goods produced in different countries, since countries have different production technologies, or differences in the price index of the baskets of goods of the two countries, as consumers may have different preferences.

[7]Note that if both economies share the same home bias, then it is not possible to define relative prices.

We can also define the terms of trade as the relation between the producer prices of both economies:

$$T_t = \frac{P_{F,t}}{P_{D,t}} \tag{29}$$

Since the economies share the same currency, the nominal exchange rate must equal one at all moments. On the other hand, the real exchange rate is given by the ratio between the two economies consumer price indexes:

$$Q_t = \frac{P_t^{cF}}{P_t^{cD}} \tag{30}$$

The real exchange rate equals one whenever countries share the same home bias in consumer preferences. Combining equations (29) and (30), we get $Q_t = (T_t)^{\varpi - \varpi^*}$.

3.4 Market equilibrium

Consider the home economy. The goods market is in equilibrium when the production of home produced goods equals home and external demand.

$$Y_t^D(i) = nC_{D,t}^D(i) + (1-n)\, C_{D,t}^F(i) \tag{31}$$

Equation (31) gives the market clearing condition for the home produced good i. The good i, produced by domestic firm i, can be consumed internally ($C_{D,t}^D(i)$) or can be exported ($C_{D,t}^F(i)$), at the same price in either case (see previous section). The share of internal consumption and exports on domestic output is given by the size of the economy for all types of goods. The consumption of the good i can be expressed in terms of total consumption of domestic goods, taking into account that there is product differentiation and that the elasticity of substitution between goods from the same country is θ.

$$C_{D,t}^D(i) = \frac{1}{n}\left(\frac{P_{D,t}(i)}{P_{D,t}}\right)^{-\theta} C_{D,t}^D \tag{32}$$

The total consumption of domestic goods can also be rewritten in terms of the total consumption index of the home economy, taking into account consumer prices and the existence of a home bias:

$$C_{D,t}^{D} = \varpi \left(\frac{P_t^{cD}}{P_{D,t}} \right) C_t^{D} \tag{33}$$

With a similar rationale, exports of the good i can also be defined in terms of the foreign total consumption index:

$$C_{D,t}^{F}(i) = \frac{1}{n} \left(\frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\theta} C_{D,t}^{F} = \frac{1}{n} \left(\frac{P_{D,t}(i)}{P_{D,t}} \right)^{-\theta} \varpi^{*} \left(\frac{P_t^{cF}}{P_{D,t}} \right) C_t^{F} \tag{34}$$

Replacing the above expressions in equation (31), and taking into account that there is free trade of goods in the whole of the area, that there is only one currency and that the law of one price holds, we get the following market clearing condition for the good i:

$$Y_t^{D}(i) = \left(\frac{P_{D,t}}{P_{D,t}(i)} \right)^{\theta} \frac{1}{P_{D,t}} \left[\varpi P_t^{cD} C_t^{D} + \frac{(1-n)}{n} \varpi^{*} P_t^{cF} C_t^{F} \right] \tag{35}$$

After some simple algebra, the aggregate domestic goods market clearing condition (per capita) results in the following equation, using the definition for the terms of trade:

$$Y_t^{D} = T_t^{(1-\varpi)} \varpi C_t^{D} + \frac{(1-n)}{n} \varpi^{*} T_t^{(1-\varpi^{*})} C_t^{F} \tag{36}$$

In a similar way, we get for the foreign economy the following market clearing condition for the good i^{*}:

$$\begin{aligned} Y_t^{F}(i^{*}) &= nC_{F,t}^{D}(i^{*}) + (1-n) C_{F,t}^{F}(i^{*}) \\ &= \left(\frac{P_{F,t}}{P_{F,t}(i^{*})} \right)^{\theta^{*}} \frac{1}{P_{F,t}} \left[(1-\varpi) \frac{n}{1-n} P_t^{cD} C_t^{D} + (1-\varpi^{*}) P_t^{cF} C_t^{F} \right] \end{aligned}$$

And for the aggregate foreign production, we have:

$$Y_t^F = T_t^{-\varpi}(1-\varpi)\frac{n}{1-n}C_t^D + (1-\varpi^*)T_t^{-\varpi^*}C_t^F \tag{37}$$

3.5 Area wide economy

The home economy and the foreign economy are linked and together they make a single currency area to which the common central bank reacts to. Thus, we have to define the area wide main conditions.

$$Y_t = \left(Y_t^D\right)^n \left(Y_t^F\right)^{(1-n)} \tag{38}$$

$$C_t = \left(C_t^D\right)^n \left(C_t^F\right)^{(1-n)} \tag{39}$$

$$L_t = \left(L_{D,t}\right)^n \left(L_{F,t}\right)^{(1-n)} \tag{40}$$

$$\pi_t = \left(\pi_t^{cD}\right)^n \left(\pi_t^{cF}\right)^{(1-n)} \tag{41}$$

Equations (38) to (41) give the main area wide variables, i.e., output, consumption, labour and consumer price inflation, respectively, as a weighted average of both economies. Consumer price inflation is defined as the rate of change of consumer prices, i.e., $\pi_t^{cD} = \frac{P_t^{cD}}{P_{t-1}^{cD}}$ and $\pi_t^{cF} = \frac{P_t^{cF}}{P_{t-1}^{cF}}$.

3.6 The log-linearized model

Given that the model shows a significant degree of nonlinearities, a straightforward solution is not available. Therefore, we follow the literature and approximate the model by log-linearizing it around the steady-state[8], which result will be used for the simulations in section 4. The variables with a hat ($\hat{}$) refer to the log-linearized variables around the steady state. We will define only country-specific shocks (except the monetary policy

[8] Appendix A presents the equations in the steady-state. Given its non-linearity, we do not present the explicit solution for the steady-state variables, but it can be proved that, given the calibrated parameters used along the paper, there is a unique and stable steady-state.

shock), since it is the most flexible option; we can easily consider area wide shocks by making the shock in one country equal to the other country's shock.

From equations (7) and (8), we get the consumption equation for the home economy[9]:

$$\hat{C}_t^D = \frac{h}{1+h}\hat{C}_{t-1}^D + \frac{1}{1+h}E_t\hat{C}_{t+1}^D + \frac{1-h}{\sigma_c(1+h)}\left(\hat{\varepsilon}_t^{bD} - E_t\hat{\varepsilon}_{t+1}^{bD}\right) - \frac{1-h}{\sigma_c(1+h)}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^{cD}\right) \tag{42}$$

Given the existence of habit formation in consumption, current consumption depends also on past consumption. The external habit parameter also influences the way consumption reacts to the interest rate and the consumer price inflation, as we would expect a lower sensitivity of current consumption to changes in the real interest rate when habit is larger. Consumption is subject to an AR(1) preference shock with an i.i.d. term ($\hat{\varepsilon}_t^{bD} = \rho_B\hat{\varepsilon}_{t-1}^{bD} + \xi_t^{bD}$).

The real wage equation is derived from equations (13) and (14):

$$\begin{aligned} \hat{w}_{D,t} \quad = \quad & \frac{\beta}{1+\beta}E_t\hat{w}_{D,t+1} + \frac{1}{1+\beta}\hat{w}_{D,t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1}^{cD} - \\ & -\frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t^{cD} + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1}^{cD} - \frac{1}{1+\beta}\frac{\left(1-\beta\xi_w^D\right)\left(1-\xi_w^D\right)}{\left(1+\varphi\sigma_L\right)\xi_w^D} \times \\ & \times\left[\hat{w}_{D,t} - \sigma_L\hat{L}_t^D - \frac{\sigma_c}{1-h}\left(\hat{C}_t^D - h\hat{C}_{t-1}^D\right) - \hat{\varepsilon}_t^{LD}\right] \end{aligned} \tag{43}$$

The real wage is set-up according to past and expected future real wage, to past, current and future consumer price inflation and according to the difference between the real wage and the one that would prevail under flexible wage setting. In case wages can not adjust every period to past inflation ($\gamma_w = 0$), then past inflation will not influence the current wage and the impact of current inflation will be smaller. The greater the wage rigidity (ξ_w^D), the labour demand elasticity (φ) and the risk aversion coefficient on labour

[9] We only present the equations for the home economy in the main text. The equations for the foreign economy are similar to the home economy and are presented in Appendix B.

supply (σ_L), the lower the reaction of current wage on optimal wage. Labour is subject to a AR(1) labour supply shock with an i.i.d. term ($\hat{\varepsilon}_t^{LD} = \rho_L \hat{\varepsilon}_{t-1}^{LD} + \xi_t^{LD}$).

From equations (23) and (24) we get the producer price inflation equation:

$$\hat{\pi}_{D,t} = \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{D,t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{D,t-1} + \frac{1}{1+\beta\gamma_p} \frac{\left(1-\beta\xi_p^D\right)\left(1-\xi_p^D\right)}{\xi_p^D} \left(\hat{w}_{D,t} - \hat{A}_t^D\right) \tag{44}$$

Producer price inflation depends on past and expected future inflation and on the current marginal cost. When the price indexation parameter γ_p is higher, current producer price inflation is more sensitive to past inflation and less sensitive to expected future inflation. The effect of marginal costs on producer price inflation depends on the price rigidity: the higher the price rigidity (ξ_p^D), the lower inflation will tend to be. The marginal cost is subject to an AR(1) productivity shock with an i.i.d. term ($\hat{A}_t^D = \rho_a \hat{A}_t^D + \hat{\eta}_{a,t}^D$).

Taking (27) and (44), we can define the equation for the consumer price inflation:

$$\hat{\pi}_t^{cD} = \varpi \hat{\pi}_{D,t} + (1-\varpi)\,\hat{\pi}_{F,t} \tag{45}$$

From (29) we define the terms of trade expression:

$$\hat{T}_t = \hat{\pi}_{F,t} - \hat{\pi}_{D,t} + \hat{T}_{t-1} \tag{46}$$

Using equation (18), we can define the home aggregate production function as:

$$\hat{Y}_t^D = \phi^D \left(\hat{A}_t^D + \hat{L}_t^D\right) \tag{47}$$

where $\phi^D = \left(1 + \frac{\Phi^D}{\bar{Y}^D}\right)$ is 1 plus the steady state proportion of home fixed costs over home output.

The market equilibrium condition is derived from (36), assuming that in the steady state the aggregate price parity between both economies holds (i.e. the terms of trade equal 1 in the steady state):

$$\hat{Y}_t^D = \hat{T}_t \left[(1-\varpi)\varpi c_y^D + (1-\varpi^*)\left(1-\varpi c_y^D\right)\right] + \varpi c_y^D \hat{C}_t^D + \left(1-\varpi c_y^D\right)\hat{C}_t^F \quad (48)$$

where c_y^D is the proportion of domestic consumption over domestic output.

The area wide main variables equations are calculated from equations (38) to (41):

$$\hat{\pi}_t = n\hat{\pi}_t^{cD} + (1-n)\,\hat{\pi}_t^{cF} \quad (49)$$

$$\hat{Y}_t = n\hat{Y}_t^D + (1-n)\,\hat{Y}_t^F \quad (50)$$

$$\hat{C}_t = n\hat{C}_t^D + (1-n)\,\hat{C}_t^F \quad (51)$$

$$\hat{L}_t = n\hat{L}_t^D + (1-n)\,\hat{L}_t^F \quad (52)$$

Given the absence of external economies to the currency area, the absence of capital and savings and given that we imposed balanced accounts in every period, we have the following aggregate equilibrium condition[10]:

$$\hat{Y}_t = \hat{C}_t \quad (53)$$

Monetary policy stance follows a similar structure to the one in Smets and Wouters (2003). The common central bank tries to stabilize the area wide inflation at the steady-state level, taking into account also the output gap. The central bank has a preference for smoothing the interest rate path and responds to changes in the short-term in inflation and output. The central bank's objective is to keep the inflation rate and the output at the steady state level. It follows the rule:

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1-\gamma_R)\left[\gamma_\pi \hat{\pi}_t + \gamma_y \hat{Y}_t\right] + \gamma_{\Delta\pi}\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + \gamma_{\Delta y}\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t \quad (54)$$

The parameters γ_π and γ_y give the weights of, respectively, inflation and output deviation from the steady state on the central bank policy rule; $\gamma_{\Delta\pi}$ and $\gamma_{\Delta y}$ weight the changes

[10] This condition will serve as a clearing condition for the area aggregation.

of inflation and output, respectively. The parameter γ_R is the interest rate persistence or the preference of the central bank for smoothing the interest rate path.

The central bank's rule also includes a money AR(1) shock with an i.i.d. term ($\hat{m}_t = \rho_m \hat{m}_{t-1} + \xi_t^m$).

4 Simulation and analysis of the results

In this section[11] we describe the behaviour of the model[12] in response to the shocks included in the model: one demand shock (the preferences shock $\hat{\varepsilon}_t^{bD}$ and $\hat{\varepsilon}_t^{bF}$), two supply shocks (the productivity shock $\hat{A}_t^D$ and $\hat{A}_t^F$, and the labour supply shock $\hat{\varepsilon}_t^{LD}$ and $\hat{\varepsilon}_t^{LF}$) and one common monetary policy shock ($\hat{m}_t$). We consider first the case of perfectly symmetric countries, i.e., both economies share the same parameters and show no home bias in consumption ($\varpi = \varpi^* = 0.5$). In this way, we intend to characterize the area wide economy and its behaviour in the presence of shocks. More precisely, we intend to approach our model to the euro area, so that we calibrate the model parameters close to the estimates found by Smets and Wouters (2003). Nonetheless, given the general characteristics of the model since it can be applied to any currency area, we assume a similar process for every shock considered in the model, which is different from Smets and Wouters (2003).

We also consider initially that shocks are common to both economies and that there are no idiosyncratic shocks. Later, we consider idiosyncratic shocks and relax the perfect symmetry hypothesis.

[11] We use Dynare running in Matlab for the simulations presented in the paper.

[12] Equation (1) in Appendix B is not included in the calibrated and simulated model. Given the aggregate equilibrium condition (53), the foreign consumption is obtained as a residual.

4.1 Homogeneous countries and common shocks

4.1.1 Calibration

As it was already mentioned, the calibration exercise intends to follow what is commonly used in recent literature on DSGE models of the euro area, namely Smets and Wouters (2003). Table 1 summarizes the calibration made in this section. Since we assume in this first exercise that economies are similar, the table presents the general parameters values.

The parameters c_y^D and c_y^F are made consistent with the steady-state levels for the calibrated parameters and ϕ^D and ϕ^F are such that the steady-state profits are null (Christiano et al., 2005).

Table 1: Parameters values used in the calibration and simulation of the model.

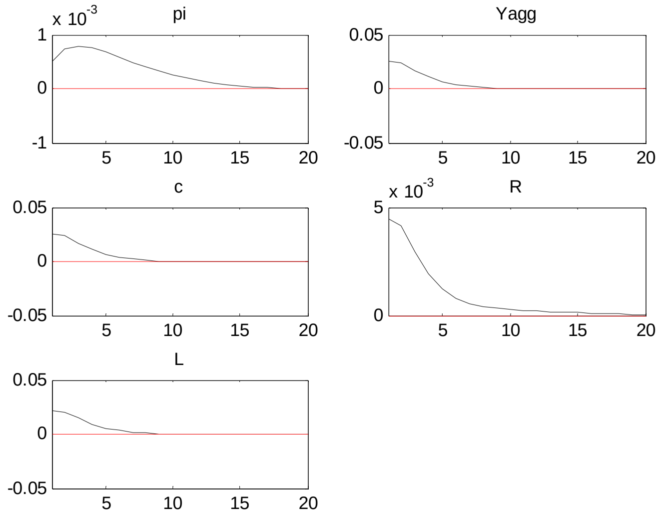
Parameter	**Description**	**Value**
β	Discount factor	0.99
h	Consumer persistence	0.6
σ_c	Relative risk aversion coefficient on consumption	1.4
ϖ	Home bias	0.5
σ_L	Relative risk aversion coefficient on labour supply	2.4
γ_w	Wage indexation	0.75
ξ_w	Calvo probability on wages	0.7
φ	Labour demand elasticity	3
γ_p	Price indexation	0.5
ξ_p	Calvo probability on prices	0.9
θ	Price elasticity	6
n	Size of the home economy	0.5
γ_R	Interest rate smoothing	0.8
γ_π	Weight on inflation	1.7
γ_y	Weight on output	0.1
$\gamma_{\Delta\pi}$	Weight on inflation differential	0.15
$\gamma_{\Delta y}$	Weight on output differential	0.15
ρ_B	Persistence of the preference shock	0.4
ρ_L	Persistence of the labour supply shock	0.4
ρ_a	Persistence of the productivity shock	0.4
ρ_m	Persistence of the monetary policy shock	0

Take note that, unlike Smets and Wouters (2003), we assume a lower persistence of shocks, which is also similar to all shocks. The persistence levels are more in line with the estimations of Jondeau and Sahuc (2006) for the preference and productivity shocks. For simplicity reasons, we considered that all shocks have the same persistence, except the monetary policy shock, since the policy rule already has persistence. Increasing the persistence of shocks does not change the qualitative results of the model, but the effects of shocks last longer. Additionally, all shocks have the same size, which is also contrary to Smets and Wouters (2003).

4.1.2 Model dynamics

Figures 2, 3, 4 and 5 show the impulse response functions (i.r.f.) of the aggregate area to the four shocks considered in the model[13]. Shocks are all simulated with the same size of 0.1. In every case, the variables return to the steady-state level in the time span of almost 5 years. Given the aggregate equilibrium condition (53), the evolution of output always equals the path for consumption.

Figure 2: Area wide variables' impulse responses to a common preference shock (ξ_t^b).

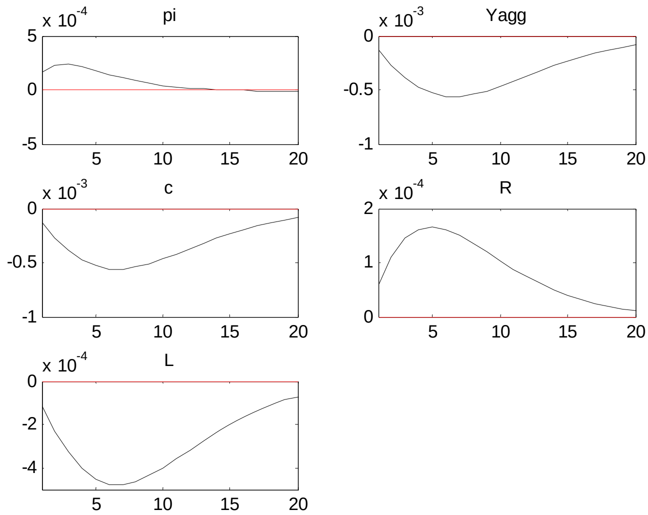


The preference shock ξ_t^b is a shock to the discount factor affecting the intertemporal substitution of households. A positive preference shock (Figure 2) increases consumption and output in the moment when the shock is observed. Consumption starts to return to the baseline in the following moment due to the increase observed in the real interest rate. The increased demand pushes up prices, although slightly, and labour and wages. Inflation

[13] The labels in the charts represent euro area aggregate variables: 'pi' for consumer price inflation, 'Yagg' for output, 'c' for consumption, 'R' for the nominal interest rate and 'L' for labour supplied. All variables are in percentage deviations from the steady state with the exception of inflation and nominal interest rate which are expressed as percentage point deviation.

deviation is small, but increases until it reaches a maximum 3 quarters after the shock was observed. The interest rate reacts immediately to the changes observed in output and inflation. However, inflation and output differential terms imply that the nominal interest rate starts to converge to the steady-state in the following period, reaching it around 20 quarters later. The observed response is broadly in line what is observed in the literature. For example, Smets and Wouters (2003) show i.r.f. with a slightly higher impact, but with a similar relative size. The main difference is that our model produces the maximum response on consumption, output and the interest rate in the first period, instead of producing a more hump-shaped response.

Figure 3: Area wide variables' impulse responses to a common labour supply shock (ξ_t^L).



The impact of a negative labour supply shock is quite moderate (Figure 3). The maximum deviation from the steady state of labour, consumption and output occurs one and a half year after the shock has been observed. The immediate increase observed in wages is reflected in the increase in area inflation. Given the sluggish behaviour in wages and prices, the maximum impact on inflation is reached 3 quarters after the shock was observed. The central bank reacts to the increasing inflation and to the changes in output. However, given the preference for smoothing the interest rate path and given that output is below the steady state level, the real interest rate $\left(\text{not shown in the figures, but equal to } \left(\hat{R}_t - \hat{\pi}_t\right)\right)$ is below the initial level during the first year. Inflation returns first to the steady state and 20 quarters after the shock was observed all variables are quite close to the steady state level. The impact of a labour supply shock in our framework leads to a weaker and slightly less persistent response regarding real variables than what has been observed in other estimated DSGE models of the euro area (e.g. Smets and Wouters, 2003).

Figure 4: Area wide variables' impulse responses to a common productivity shock ($\eta_{a,t}$).

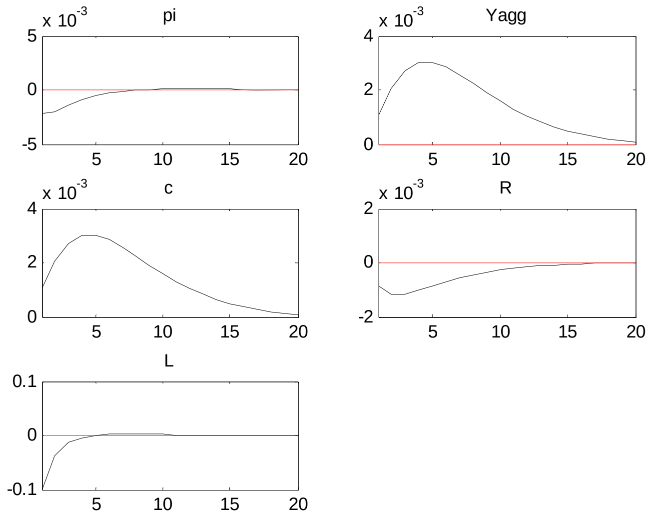
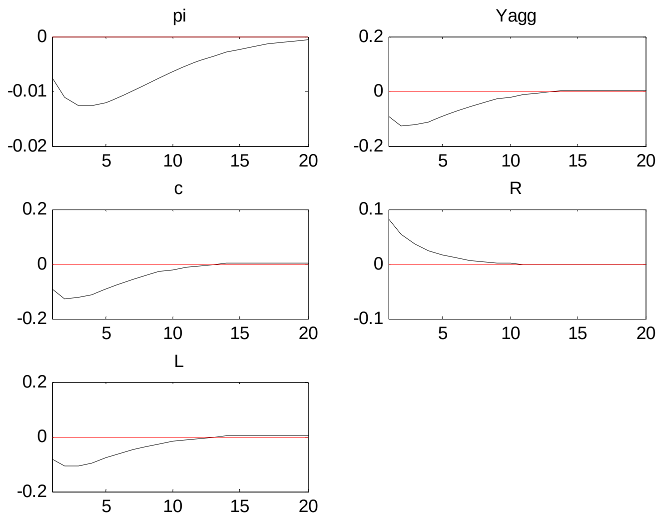


Figure 4 shows the i.r.f. after a positive technology shock was observed. A technology shock has as a consequence an increase in productivity, as firms can produce the same amount of goods at a lower cost. Thus, we observe an immediate decrease of marginal costs, allowing for the observed increase in output and decrease in prices, which then stimulates consumption. Firms can now produce the same amount of goods with lower input, implying a relevant decrease in labour supply. However, labour quickly returns to the steady state level at the end of the first year. Monetary policy reacts with some delay to the fall in inflation, given the high preference for smoothing the interest rate path.

The response of the model to the productivity shock is also partly in line with the literature. As we concluded regarding the preference and labour supply shock, our model seems to imply a slightly lower impact and persistence of shocks. Monetary policy responds less to the shock, allowing for a positive real interest rate during the first periods, while in Smets and Wouters (2003) we observe a negative impact on the real interest rate. Also the impact on wages in our model is more pronounced, while in Smets and Wouters (2003) they remained almost unchanged.

Figure 5: Area wide variales' impulse responses to a common monetary policy shock (ξ_t^m).



Finally, figure 5 shows the impact of a temporary monetary policy tightening shock. This generates a strong response in a hump-shaped pattern in all variables of the model. The maximum effect is observed during the first year. The higher interest rate motivates consumers to postpone consumption, leading also to the decrease in output in order to respond to the lower demand. Given this, firms need less labour. Nonetheless, at the end of the first year, the economy starts to recover. This is a quicker recovery than what is generally observed in literature (Smets and Wouters, 2003), which identifies a year and a half for the maximum effects to be felt on the same variables as considered here. Output, consumption, labour and the interest rate return to the steady state around 3 years after the shock was observed, while the sluggish behaviour of prices only allows for the return of inflation to the steady state one year later. This is also an earlier return to steady state when compared to Smets and Wouters (2003).[14]

4.2 Heterogeneous countries

As was seen in section 2, euro area countries are not completely homogeneous and we observe persistent differentials in some macro variables. Differences can be due to the convergence process between member countries. But the justification can also come from differences in structural features of the countries. Then, how important are these differences when countries do not have independent monetary policy, which responds only to the aggregate?

This section displays the reaction of the economies of our model and the area wide economy when subject to shocks, common and idiosyncratic, taking into account that we will change some parameters of the domestic economy and leave unchanged the foreign economy from the baseline (symmetric) case. The shocks considered are the following: two common shocks (a shock to consumer preferences[15] and the monetary policy shock) and two country specific shocks (labour supply shock and technology shock). We will mainly focus on what happens when the domestic economy has different wage and price mark-ups and rigidities from those in the foreign country and when domestic consumers have different preferences for domestically produced goods. First, the response of the economies under the same monetary policy rule considered before is analyzed, and afterwards we will

[14] The faster adjustment in our model than in Smets and Wouters (2003) can be due to the lower shock persistence of shocks in our model. The fact that Smets and Wouters (2003) model also includes capital and further rigidities can also contribute to the delayed responses.

[15] The foreign consumption is determined by default (see footnote 6). In this way, we can only consider a preference shock to the home economy, which will work out as a common shock.

also focus on the impact of different monetary policy rules. Finally, we intend to perform a welfare analysis on the different scenarios studied.

It should be reminded that we abstract from fiscal policy issues, although acknowledging that when fiscal policy is taken into account, the negative impacts of asymmetric countries or asymmetric shocks in a currency union are less relevant (Adão et al., 2006).

4.2.1 Model dynamics

Heterogeneity regarding the home bias

We consider in this section that the parameters are the same as the ones in Table 1, with the exception of the home bias of the domestic economy consumers. Therefore, we consider $\varpi = 0.8$, i.e., domestic households prefer to consume home produced goods and these represent 80% of the average consumption basket.

The dynamics of the model following a common shock (preferences or monetary policy) is similar to what occurs when the economies are symmetric. Therefore, heterogeneity regarding the home bias does not lead to heterogeneity in the response to common shocks (see figure 13 in Appendix C).

On the other hand, in comparison to the baseline scenario, country specific shocks (labour and technology) impact differently the economies and this response is affected by the existence of a home bias (see figure 13). These shocks have a greater impact in the country where they are observed. Comparing to the baseline scenario (figure 12), we observe larger deviations from the steady state in the home production after home-specific shocks. Thus, countries with a larger home bias are slightly more affected by specific shocks.

The response to country-specific shocks of consumer price inflation is different from the baseline scenario and we observe inflation differentials between the two countries. Indeed, only when there is heterogeneity regarding the home bias we observe different responses of inflation and consumption between countries. The country that was hit by the shock shows a more significant impact on its consumer price inflation. We also observe a slightly larger impact on aggregate inflation when the shock occurs at home, leading to a harsher response of monetary policy in the first quarters following the shock. The impact in consumption in the country hit by the shock is smoother when this is the country with higher home bias.

Monetary policy responds in a similar way as in the baseline scenario, but the impact on the interest rate is more persistent when the country-specific shock occurs in the economy with a lower bias (see figure 13).

Heterogeneity regarding wage and price mark-up

The wage mark-up is given by $\dfrac{\varphi}{\varphi - 1}$. In the baseline scenario, both economies shared a wage mark-up of 1.5. In the current section, it is assumed that the labour demand elasticity φ in the home economy diminishes so that the wage mark-up increases to 2, while the foreign economy is unchanged from the baseline scenario. As regards the price mark-up, this is given by $\dfrac{\theta}{\theta - 1}$, and θ is subject to a similar interpretation as for φ. We also consider a higher price mark-up in the home economy, equal to 1.67 (vs. 1.2 in the baseline scenario).

We don not observe significantly different i.r.f. when the home economy price mark-up differs from the rest of the union (see figures 12 and 16). We only notice a slight decrease in the size of the responses of labour, due to the rise in the fixed costs.

The i.r.f. for the case with different wage mark-up for the home economy are quite close to the baseline scenario of homogeneous countries (see figures 14). In the case of the common shocks (preference or monetary policy), we observe a very slightly more pronounced impact in inflation. Terms of trade become favourable for the home economy following a positive common shock because the higher increase in wages due to the higher mark-up pushes up producer prices more than in the foreign economy.

The i.r.f. following shocks specific to the foreign country (which maintained its wage mark-up) (figures 14 and 16) are similar to the symmetric countries case (figure 12), either for the labour supply or the productivity shock. When the shock occurs in the home country, the impact is slightly higher than in the homogeneous countries case, (see figure 14), due to the higher impact on wages.

All in all, the main conclusion is that wage and price mark-up differences among countries do not lead to significantly different responses to shocks of the economies when comparing to the baseline scenario.

Heterogeneity regarding nominal rigidities

Wage rigidity Recall that wages are set according to a Calvo mechanism. Every period, agents face a constant probability $1-\xi_w$ of optimizing wages. With a probability ξ_w, agents can not reoptimize and adjust wages in a proportion of the past period consumer price inflation. In the baseline scenario, we assumed $\xi_w^D = \xi_w^F = 0.7$. Now, consider that the wage setting in the home economy is even more sluggish and agents face a probability of optimizing wages at date t of only 1% ($\xi_w^D = 0.99$). It is expected that this high degree of wage rigidity will induce a slower adjustment after shocks, mainly in the more rigid country. Abbritti (2007) finds out that shocks tend to have larger real effects in the more sluggish country and this is more pronounced when the shocks hit this country. He also notes that the size and persistence of inflation and unemployment differentials increases with the degree of wage rigidity. The differences occur even in face of common shocks.

The results of the simulation are presented in figure 15. As can be seen, both common and specific shocks have a different impact on the two countries when the home economy has a higher wage rigidity and, broadly, economies response is smoother and more persistent than in the baseline specification.

Starting with the analysis of the common shocks, these lead to a broadly similar response of the economies in comparison to the baseline scenario, although the effects of the shocks last longer, since it takes longer for all domestic households to adjust their wages to the steady state level. Wage adjustment is faster and stronger in the foreign country, with a larger impact on prices. In comparison to the baseline, a positive preference shock implies an increase in demand and this leads to unfavorable terms of trade to the home economy, since producer prices increase at a slower pace. The inverse occurs for the monetary policy shock.

Regarding specific shocks, supposing there is a shock in the more sluggish country, the impact in this country is smoother and much more persistent than in the baseline scenario, especially after a labour supply shock (see figure 15). This is due to the high wage rigidity, which implies a small reaction of the economy to the shock. When the home economy is hit by a labour supply shock, the reduction in labour and output is smoother than in the baseline but it takes longer to reach its maximum effect (around 13 quarters vs 7 quarters in the baseline). The increase in inflation due the increased production costs is also smoother and lasts longer, which justifies the small and slow response of monetary policy, with the interest rate reaching the maximum deviation 3 years after the shock was observed.

In case the labour supply shock occurs in the foreign country, the impact is closer to the baseline scenario. It is worth mentioning that due to the high wage rigidity in the home economy, production costs do not change much in the home economy after the shock. Thus, terms of trade are more favourable to the foreign economy and more than in the baseline, which justifies the increase in home output.

When the home economy is hit by the productivity shock, this does not entirely benefit the workers' income since wages are not able to adjust easily in response to the increased productivity. The rise of home productivity increases demand for domestically produced goods by comparison with foreign goods, reducing foreign output and labour.

The disadvantage in terms of productivity in the foreign country is compensated by wage adjustment. The expansionary monetary policy also contributes to the foreign country recovery. At a first phase, terms of trade become more unfavorable to the home economy, given the lower production costs in the home economy. The higher flexibility in the foreign country allows for a better recovery in this country, while the home economy will have its economic activity decreasing below the initial level.

When shocks occur in the more flexible country, the response of the macro-variables is similar to the baseline scenario, although less pronounced in the foreign country (see figure 15). This is because the higher wage rigidity in the home country has a lower impact in consumer price inflation, and then, the impacts on demand and output are smaller.

Price rigidity Prices are set à la Calvo, similarly to the wage setting. In this way, ξ_p is the probability of firms not optimizing prices at each moment. In this section, we assume that ξ_p is lower in the home economy, i.e., prices are more flexible in the home economy than in the foreign economy. More specifically, we set $\xi_p^D = 0.09$, which implies a relatively high probability of domestic firms being able to optimize prices. It is expected that the more flexible country regarding price setting reacts more and faster to shocks. Gomes (2004) studies the effects of common and specific shocks in countries with different degrees of price rigidity and amid different monetary policy rules. Common shocks lead to different responses from countries and are more favourable to the less rigid country. A technology shock to the more flexible country increases the inflation and output differentials and favours the less rigid country. When the technology shock hits the more price sluggish country, the responses are smoother in both countries and in the union, with an almost negligible effect in the more flexible country. Monetary policy reacts to the monetary union variables, which are controlled by the effects in the country hit by the shock. Differentials are lower when the central bank has a preference for smoothing the interest rate.

In our model, the response of the area wide variables following common shocks is relatively similar to the symmetric countries case (see figure 17 in comparison to figure 12). However, variables for the home economy return faster to steady-state than in the symmetric case, because domestic agents change prices more rapidly. Aggregate inflation has a sharper deviation from the steady state level than in the baseline scenario, but it reaches its maximum effect around two or three quarters after the shock, one quarter earlier than in the baseline, and it also returns earlier to the steady state level.

Regarding country-specific shocks, when these occur in the more price flexible country, i.r.f. in both countries are more pronounced and return faster to the steady state level than in the baseline scenario; when shocks occur in the more rigid country, the response is slightly less pronounced and the hump is reached slightly later although it takes the same time to return to the steady-state (see figure 17).

When the labour supply shock occurs in the home economy, the reaction of prices is larger than in the baseline, which increase in response to the higher costs of production. Consumer price inflation increases, decreasing demand of domestically produced goods. The increase in producer price inflation in the foreign economy is lower than at home, given the higher price rigidity. Thus, terms of trade improve in the home country. Nonetheless, the high flexibility allows the home economy to rapidly recover and return to the initial steady state level. Monetary policy responds to the rise in consumer price inflation whose evolution is dominated by the evolution in home producer price inflation.

When the labour supply shock occurs in the foreign country, the immediate reaction is similar to when it occurs at home. However, the price reaction is more moderate. Domestic producers rapidly adjust their prices downwards in order to face the lower aggregate demand. This gives a competitive advantage to home producers (positive terms of trade). Given that the producer price inflation in the foreign economy and at home follows inverse directions, consumer price inflation does not deviate much from the steady state and, hence, the monetary policy reaction is very low.

A technology shock in the home economy increases productivity, allowing for an immediate and strong downwards adjustment in home goods prices. Foreign producer price inflation also decreases, but more smoothly. Consequently, consumer price inflation decreases and we observe an expansionary monetary policy which again stimulates demand.

If the technology shock had occurred in the foreign country, the downward shift in inflation would be much more modest and persistent in comparison to the case when it occurs in the home country and to the baseline scenario, as foreign producer price inflation would decrease less and home producer price inflation would rise as it had no productivity gains relatively to the foreign country. Therefore, interest rate decreases, given the fall in inflation and the positive deviation in output. At home, the immediate impact of the decrease in output is swiftly adjusted through producer prices.

Summing up, the main results from the analysis developed above are the following: (i) there are only consumer price inflation differentials when the home bias in consumer preferences differs between countries; (ii) different levels of the wage or price mark-up do not imply significantly different responses of the economies to shocks; (iii) higher wage or price rigidity lowers the speed of adjustment after shocks and the degree of rigidity of the country hit by specific shocks determines the rhythm of the return of both economies to the steady state: when a specific shock occurs in the more sluggish country, both economies take longer to return to the steady state than when the shock occurs in the more flexible country.

4.2.2 Different monetary policy rules

In the previous sections, we have seen that the model is able to replicate the most important results present in the literature and namely the response of the euro area economy to shocks. The reaction of the two economies to common and specific shocks, when they diverge regarding home bias in consumption preferences and wage or price setting features, has also been reviewed. This analysis was done taking into account only one monetary policy rule, a kind of expanded Taylor rule, with features recently used in literature, such as interest rate smoothing and differential component of inflation and output gap (Smets and Wouters, 2003).

We are now interested in analyzing the volatility of the main variables of each country in response to the shocks included in the model considering different monetary policy rules. Specifically, we consider the original rule and seven derivations from this rule:

1. Original rule: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(1.7\hat{\pi}_t + 0.1\hat{Y}_t\right) + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

2. Rule without differential component: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(1.7\hat{\pi}_t + 0.1\hat{Y}_t\right) + \hat{m}_t$

3. Rule without smoothing: $\hat{R}_t = 1.7\hat{\pi}_t + 0.1\hat{Y}_t + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

4. Basic Taylor rule: $\hat{R}_t = 1.7\hat{\pi}_t + 0.1\hat{Y}_t + \hat{m}_t$

5. Low weight on inflation: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(\hat{\pi}_t + 0.1\hat{Y}_t\right) + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

6. High weight on inflation: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(2\hat{\pi}_t + 0.1\hat{Y}_t\right) + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

7. No weight on output gap: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(1.7\hat{\pi}_t\right) + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

8. High weight on output gap: $\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2\left(1.7\hat{\pi}_t + \hat{Y}_t\right) + 0.15\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + 0.15\left(\hat{Y}_t - \hat{Y}_{t-1}\right) + \hat{m}_t$

One can assume the first four rules as different rules that the central bank can follow, while the last four rules are the same rule but with the purpose of assessing the relevance of the different weights on inflation and the output gap.

Figures 6, 7 and 8 show the broad volatility of the economies, i.e., they display the variance of the main variables (inflation, output, interest rate, labour and terms of trade) of each economy and the aggregate, when all model shocks are in place, results which are taken after 10 000 simulations.[16]

First, the impact of changes in the parameters of the home economy in the volatility pattern of the variables is assessed. Changing the home bias (ϖ) in domestic consumption preferences does not imply relevant changes in the variables' variances, except for the terms of trade (see figure 6). Terms of trade variance is lower the lower the differences in consumer preferences, i.e., when ϖ is closer to 0.5, the same home bias as in the foreign country. This suggests that when the preference for domestic goods is the same as for foreign goods, the demand of goods produced in the two countries remains relatively balanced, and relative producer prices between the countries change less.

Changes to the home wage mark-up (φ) imply changes in the volatility pattern of some variables. Home output and labour variances increase slightly when the wage mark-up is lower, meaning that a lower mark-up leads to a slightly more reactive labour market and, consequently, production. The opposite occurs in the foreign economy, meaning that, for the union, introducing heterogeneity in the wage mark-up does not impact significantly on real variables. The inflation variance increases with the home wage mark-up, meaning that the larger the wage mark-up, the larger the reaction of prices to shocks. Take note that for all different parameters considered, home, foreign and aggregate inflation share the same variance.

[16]One should take into consideration that the following caveat applies to the analysis and comparison of rules developed in this section: the ranking between the different policy rules depends of the size and persistence of the shocks, so that if we change the calibration regarding shocks we could have a different rank of the policy rules.

Considering different values for the domestic price mark-up (θ) leads to changes in labour and output volatility. A lower price mark-up means that the share of the fixed costs in steady-state is lower. Then, maintaining the same level of output implies that labour costs can adjust more and, therefor, labour can be more volatile.

On the other hand, when wages are more rigid (higher ξ_w^D), inflation volatility decreases, as the more sluggish behaviour of wages implies less and lower changes in production costs, and therefore, a lower need to change prices. Labour and output variance share a similar pattern: the variance for the aggregate variables increases when domestic wages are more rigid, which is due to the pattern observed in the home economy, while the behaviour in the foreign economy is the opposite. Therefore, there is a higher volatility in real variables in the country with relative higher wage rigidity, meaning a higher adjustment after shocks through real variables. The same volatility pattern in inflation, labour and output is found when ξ_p^D is changed.

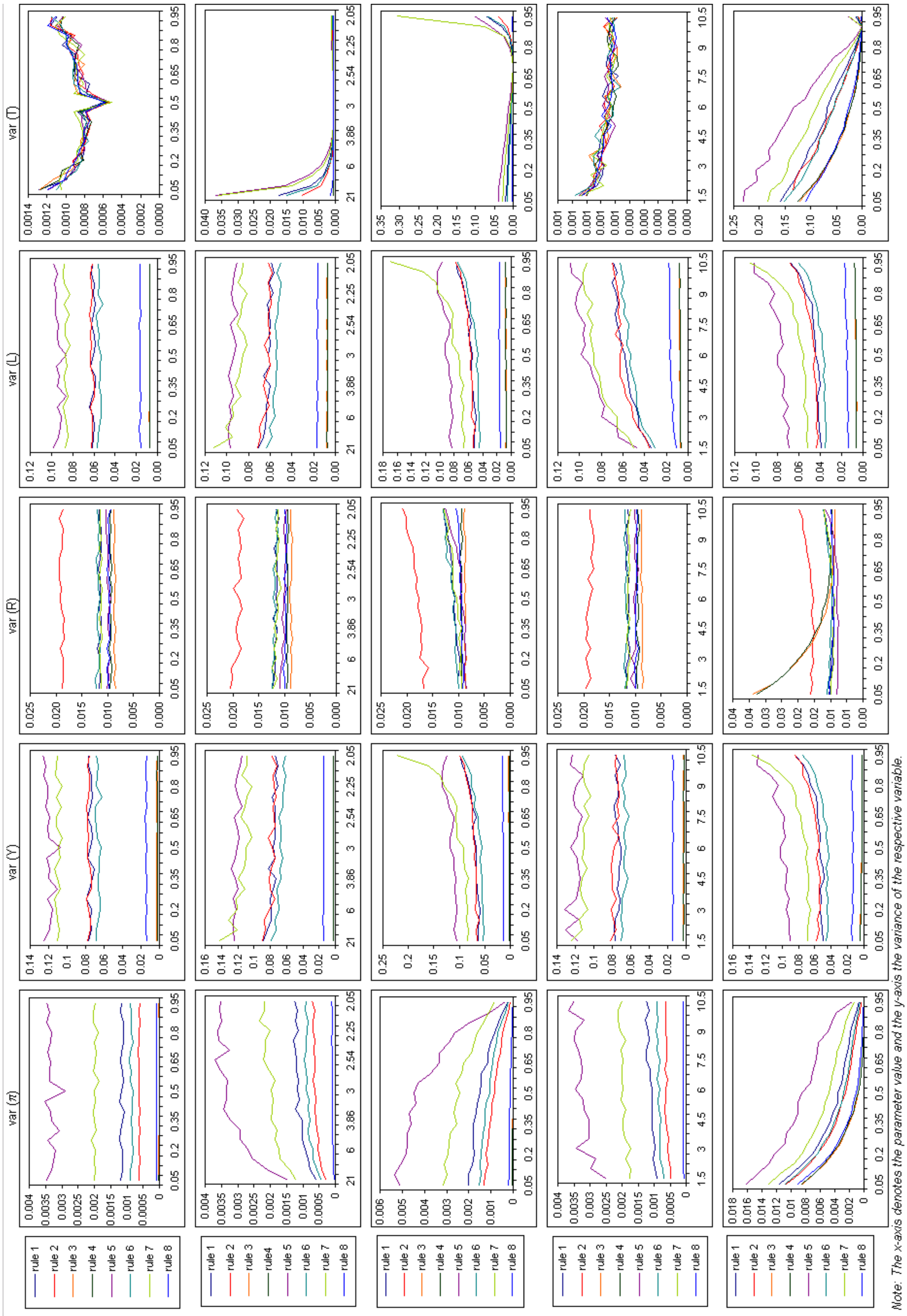
As for interest rate volatility, it increases slightly with the rigidity in wages for most policy rules considered, denoting that the interest rate has to react more when wages are more rigid. When changing the price Calvo probability in the home economy, we also denote a slight increase in volatility for the higher values of ξ_p^D, except for the rules without the smoothing term. For these rules, interest rate variance is decreasing with the price Calvo probability, reaching significantly high values when the home economy is almost fully price-flexible. This means that when prices are flexible they get more responsive to shocks and then more volatile. When the central bank does not smooth the interest rate path, the interest rate will change whenever the price evolution goes beyond the central bank objective, and then it is also more volatile.

Terms of trade variance shows a U-shape pattern for changes in nominal rigidity parameters, reaching a minimum when countries are identical.

Turning to the comparison of different policy rules, it was already mentioned that inflation volatility is the same for both countries and, therefore, the aggregate. The policy rule with a low weight on inflation is the one that generates more volatility, for all variables except the interest rate. Curiously, the rule which does not account for the output gap leads to the second more volatile inflation response. The rules without smoothing are responsible for the most stable inflation response to shocks, probably meaning that a more aggressive central bank is able to better control inflation developments. The same ranking of policy rules applies if we analyze output and labour stability.

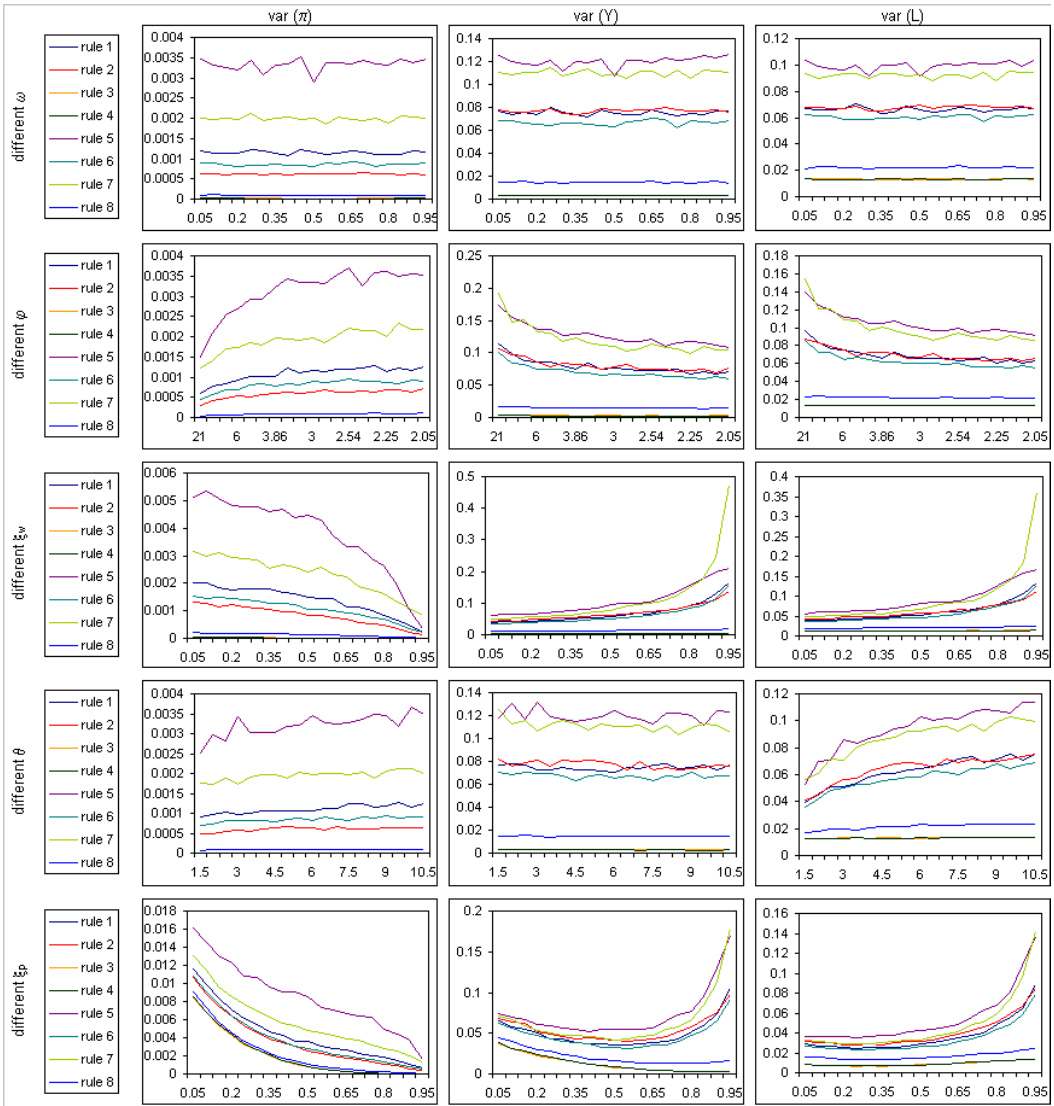
As regards the interest rate, the rule without the differential component generates more volatility, while the rules without a smoothing component result in a more stable evolution of the interest rate, similarly to what is found for the real variables and inflation. This a somehow odd conclusion, which can have as a possible explanation the fact that rules without smoothing permit a more immediate and stronger response of the central bank to shocks, implying, therefore, an overall more stable economy. There is an exception to these conclusions: the volatility of the interest rate under rules without smoothing is very high and above every other rule when the home economy is more price flexible (when ξ_p^D is small).

Figure 6: Area wide variables' volatility considering different monetary policy rules and different parameter values.



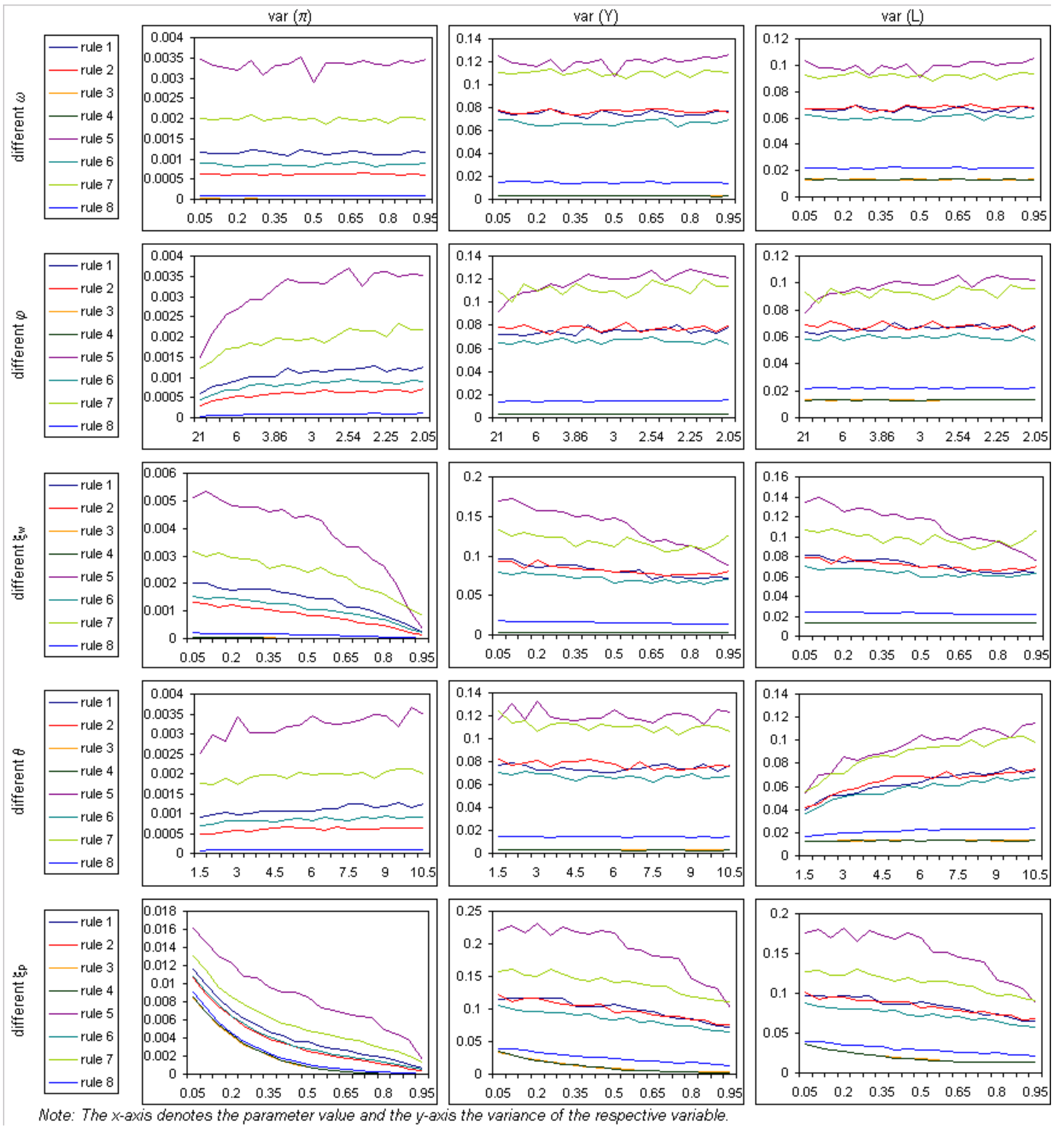
Note: The x-axis denotes the parameter value and the y-axis the variance of the respective variable.

Figure 7: Home economy variables' volatility considering different monetary policy rules and different parameter values.



Note: The x-axis denotes the parameter value and the y-axis the variance of the respective variable.

Figure 8: Foreign economy variables' volatility considering different monetary policy rules and different parameter values.



4.2.3 Welfare analysis

We have seen in the previous section that different monetary policy rules have different effects on each country's macroeconomic volatility. Volatility of the main macroeconomic variables is a core aspect behind societies welfare. Agents dislike large or rapid changes in the income. In this way, it is also important to understand how the different rules affect each country welfare.

The welfare function of each country is determined by the average utility of the representative household at moment t. First, rewrite equation (2) so that we can separate the utility function into consumption and labour:

$$\underbrace{\frac{\varepsilon_t^b}{1-\sigma_c}\left(C_t - H_t\right)^{1-\sigma_c}}_{U^D(C_t^D, H_t^D, \varepsilon_t^{bD})} - \underbrace{\frac{\varepsilon_t^b \varepsilon_t^L}{1+\sigma_l} L_t\left(i\right)^{1+\sigma_l}}_{V^D(L_t^D(j), \varepsilon_t^{bD}, \varepsilon_t^{LD})}$$

In this way, $U(.)$ denotes the utility from consumption and $V(.)$ is the disutility from working. Then, the welfare function is given by (for home and foreign economies, respectively)

$$\mathcal{W}_t^D = U^D(C_t^D, H_t^D, \varepsilon_t^{bD}) - \frac{1}{n}\int_0^n V^D(L_t^D\left(j\right), \varepsilon_t^{bD}, \varepsilon_t^{LD})dj \tag{55}$$

$$\mathcal{W}_t^F = U^F(C_t^F, H_t^F, \varepsilon_t^{bF}) - \frac{1}{1-n}\int_n^1 V^F(L_t^F\left(j^*\right), \varepsilon_t^{bF}, \varepsilon_t^{LF})dj^* \tag{56}$$

Expression (55) can be approximated by a second-order Taylor series expansion (see appendix D for details):

$$\begin{aligned}
\mathcal{W}_t^D \approx\; & \bar{\mathcal{W}}^D\left(\bar{C}^D, \bar{H}^D, \bar{L}^D\right) + \bar{\mathcal{W}}_{C^D}\tilde{C}_t^D + \bar{\mathcal{W}}_{H^D}\tilde{H}_t^D + \bar{\mathcal{W}}_{L^D}\tilde{L}_t^D + \bar{\mathcal{W}}_{\varepsilon^{bD}}\tilde{\varepsilon}_t^{bD} + \bar{\mathcal{W}}_{\varepsilon^{LD}}\tilde{\varepsilon}_t^{LD} + \\
& + \tfrac{1}{2}\bar{\mathcal{W}}_{C^D C^D}\;{\tilde{C}_t^D}^2 + \tfrac{1}{2}\bar{\mathcal{W}}_{H^D H^D}\;{\tilde{H}_t^D}^2 + \tfrac{1}{2}\bar{\mathcal{W}}_{L^D L^D}\;{\tilde{L}_t^D}^2 + \tfrac{1}{2}\bar{\mathcal{W}}_{\varepsilon^{bD}\varepsilon^{bD}}\;{\tilde{\varepsilon}_t^{bD}}^2 + \\
& + \tfrac{1}{2}\bar{\mathcal{W}}_{\varepsilon^{LD}\varepsilon^{LD}}\;{\tilde{\varepsilon}_t^{LD}}^2 + \bar{\mathcal{W}}_{C^D H^D}\tilde{C}_t^D\tilde{H}_t^D + \bar{\mathcal{W}}_{C^D L^D}\tilde{C}_t^D\tilde{L}_t^D + \bar{\mathcal{W}}_{C^D\varepsilon^{bD}}\tilde{C}_t^D\tilde{\varepsilon}_t^{bD} + \\
& + \bar{\mathcal{W}}_{C^D\varepsilon^{LD}}\tilde{C}_t^D\tilde{\varepsilon}_t^{LD} + \bar{\mathcal{W}}_{H^D L^D}\tilde{H}_t^D\tilde{L}_t^D + \bar{\mathcal{W}}_{H^D\varepsilon^{bD}}\tilde{H}_t^D\tilde{\varepsilon}_t^{bD} + \bar{\mathcal{W}}_{H^D\varepsilon^{LD}}\tilde{H}_t^D\tilde{\varepsilon}_t^{LD} +
\end{aligned}$$

$$+\bar{\mathcal{W}}_{L^D\varepsilon^{bD}}\tilde{L}_t^D\tilde{\varepsilon}_t^{bD}+\bar{\mathcal{W}}_{L^D\varepsilon^{LD}}\tilde{L}_t^D\tilde{\varepsilon}_t^{LD}+\bar{\mathcal{W}}_{\varepsilon^{bD}\varepsilon^{LD}}\tilde{\varepsilon}_t^{bD}\tilde{\varepsilon}_t^{LD}$$

where a variable with a til means its deviation from the steady state ($\tilde{x}_t = x_t - x$), $\bar{\mathcal{W}}_x$ means the first derivative of the welfare function in the steady state with respect to variable x, and $\bar{\mathcal{W}}_{xx}$ means the second derivative. This expression is derived so that the welfare function of the home economy is approximate to the following expression, which depends of variables denoted in terms of log-deviations from the steady state at moment t:

$$\mathcal{W}_t^D = \bar{U}^D\left(C^D\right)+\bar{U}_{C^D}\left(C^D\right)C^D\begin{bmatrix}\left(\hat{C}_t^D - h\hat{C}_{t-1}^D\right)+\frac{1}{2}\left(\hat{C}_t^{D\,2} - h^2\,\hat{C}_{t-1}^{D\,2}\right)- \\ -\frac{\sigma_c}{2\left(1-h\right)}\left(\hat{C}_t^D - h^2\hat{C}_{t-1}^D\right)^2+\frac{1-h}{2\left(1-\sigma_c\right)}\,\hat{\varepsilon}_t^{b\,2}+ \\ +\hat{\varepsilon}_t^{bD}\left(\hat{C}_t^D - h^2\hat{C}_{t-1}^D\right)- \\ -u_1\left(\hat{\pi}_{w,t}^D-\gamma_w\hat{\pi}_{D,t-1}\right)^2-u_2\left(\hat{\pi}_{D,t}-\gamma_p\hat{\pi}_{D,t-1}\right)^2- \\ -u_3\,\hat{Y}_t^{D\,2}+u_4\hat{Y}_t^D\hat{A}_t^D-u_5\hat{Y}_t^D\left(1+\hat{\varepsilon}_t^{bD}+\hat{\varepsilon}_t^{LD}\right)\end{bmatrix} \tag{57}$$

where:

$$\hat{\pi}_{w,t}^D = \hat{w}_{D,t}-\hat{w}_{D,t-1}$$
$$u_1 = \frac{\xi_w^D\varphi\left(\varphi\sigma_L+1\right)\left(1-\Theta\right)}{2\left(1-\xi_w^D\right)\left(1-\beta\xi_w^D\right)}$$
$$u_2 = \frac{\xi_p^D\theta\left(1-\Theta\right)}{2\left(1-\xi_p^D\right)\left(1-\beta\xi_p^D\right)\left(1+n\phi\right)}$$
$$u_3 = \frac{\left(1-\Theta\right)\left(1+n\phi+\sigma_L\right)}{2\left(1+n\phi\right)^2}$$
$$u_4 = \frac{\left(1-\Theta\right)\left(1+n\phi-\sigma_L\right)}{\left(1+n\phi\right)^2}$$
$$u_5 = \frac{1-\Theta}{1+n\phi}$$
$$\Theta = 1-\frac{\theta-1}{\theta}\frac{\varphi-1}{\varphi}$$

Similarly, we have for the foreign economy:

$$\mathcal{W}_t^F = \bar{U}^F\left(C^F\right)+\bar{U}_{C^F}\left(C^F\right)C^F\left[\begin{array}{c}\left(\hat{C}_t^F - h^*\hat{C}_{t-1}^F\right)+\frac{1}{2}\left(\hat{C}_t^{F\,2} - h^{*\,2}\,\hat{C}_{t-1}^{F\,\,2}\right)- \\ -\frac{\sigma_c^*}{2\left(1-h^*\right)}\left(\hat{C}_t^F - h^{*\,2}\,\hat{C}_{t-1}^F\right)^2+\hat{\varepsilon}_t^{bF}\left(\hat{C}_t^F - h^{*\,2}\,\hat{C}_{t-1}^F\right)- \\ -u_1^*\left(\hat{\pi}_{w,t}^F - \gamma_w^*\hat{\pi}_{F,t-1}\right)^2 - u_2^*\left(\hat{\pi}_{F,t} - \gamma_p^*\hat{\pi}_{F,t-1}\right)^2 - \\ -u_3^*\,\hat{Y}_t^{F\,2} + u_4^*\hat{Y}_t^F\hat{A}_t^F - u_5^*\hat{Y}_t^F\left(1+\hat{\varepsilon}_t^{bF}+\hat{\varepsilon}_t^{LF}\right)\end{array}\right] \tag{58}$$

The area wide welfare will be given by the weighted average of both countries welfare:

$$\mathcal{W}_t = n\mathcal{W}_t^D + (1-n)\,\mathcal{W}_t^F \tag{59}$$

The first term in brackets in the welfare function derived above shows that agents appreciate higher consumption, as long as it is at least equal to the habit term. i.e., consumers want to maintain a percentage of their previous consumption unchanged. Whenever they have the possibility to increase consumption above this habit level, they have welfare gains; when they are not able to maintain consumption at least at the habit level, they have welfare losses. Welfare also depends positively on the correlation between consumption and the preference shock. Whenever a positive preference shock induces an increase in consumption, this is valued positively by agents and there are welfare gains.

On the other hand, agents dislike rapid and large changes, i.e., volatility in their consumption level or in the ability to purchase. Volatility in the present consumption level discounted by the habit decreases welfare $\left(\frac{\sigma_c}{2\left(1-h\right)} > 0\right)$. Volatility in wage and price inflation also leads to welfare losses ($u_1 > 0$ and $u_2 > 0$). However, since there is wage/price rigidity and the agents that are not able to reoptimize can have their wage/prices reviewed by indexing to previous period producer price inflation, then the more imperfect the indexation (i.e., the larger the difference between the wages/prices optimal and indexed) the larger the welfare losses will be. Output volatility also counts negatively to the welfare, as agents prefer a smooth income path ($u_3 > 0$). Regarding the correlation of shocks with the output, this usually impacts negatively in the agents welfare ($u_4 < 0$ if $1 + n\phi < \sigma_L$ and $u_5 > 0$). There is however one exception: in case $1 + n\phi > \sigma_L$, a positive correlation between the technology shock and output is deemed as positive. The elasticity of work effort with respect to real wage must be sufficiently low in order to compensate a positive correlation between output and the technology shock[17].

Figure 9 below presents the results for the average welfare after simulations of 20 000 periods, considering different sources of heterogeneity among countries. An assessment of how different monetary policy rules affect welfare is also conducted. The first column of charts shows the results for the aggregate area, the second column the results for the home economy and the third for the foreign economy.

Recall first that the welfare expression depends of the steady-state of the economies and the reaction to shocks. Changing parameter values, namely ϖ, φ and θ, changes the steady-state welfare level, while changes in nominal rigidity parameters keep the steady-state unchanged. Therefore, the impacts on welfare as presented on figure 9 show both effects, the shift in the steady-state and the reaction to shocks.

Looking only to the impact of changing parameters in the steady-state, it is possible to conclude that: (i) increasing the home bias at home ϖ increases utility at home at decreases abroad, which combined means that union utility has a U-shape with a minimum when there is no home bias ($\varpi = \varpi^* = 0.5$); (ii) increasing the wage or price mark-ups at home increases area wide utility, which is the combined effect from an increase in utility in the home economy and a decrease in the foreign country.

From the analysis of the term regarding the log-deviations from the steady-state in the welfare expression only, it is possible to conclude that a change in wage and price mark-ups and rigidity impacts on welfare through two channels: (i) via the parameters in the welfare function, which weight agents' preferences regarding consumption, output and

[17] In our baseline scenario (Table 1) this case of sufficiently low labour elasticity does not verify and we have $u_4 < 0$.

inflation stability, and (ii) via the volatility of these variables, which change according to the structure of the economy and the monetary policy rules in practice, as was observed in the previous section. A decrease in the probability of being able to reoptimize wages or prices (ξ_w or ξ_p) increases the value of parameters u_1 or u_2 which weight the volatility of wage or price inflation in the welfare. Therefore, a lower nominal rigidity impacts positively on the welfare through the lower sensitivity of agents to inflation volatility. On the other hand, it increases inflation volatility in both countries and output volatility in the more rigid country (see section 4.2.2), which impacts negatively on welfare.

Overall, higher flexibility (either on wages or prices) in the home economy decreases welfare in both countries, which means that the second "volatility channel" dominates (as there are no changes in the steady-state level). When wages or prices are more flexible at home, welfare is lower in this country than in the foreign country. Given that nominal rigidities have a significant impact in countries welfare, a deeper analysis is performed in the next subsection.

Increases in the wage or price mark-ups at home increase the area wide welfare, through the increase in the home economy. The welfare in the foreign country, which parameters remain unchanged, decreases when the mark-ups at home rise. This effect is almost exclusively due to the impact of heterogeneity in the steady-state welfare level. In addition, despite the higher inflation volatility and the higher degree of overall inefficiency in the economy (given by parameter Θ) when the mark-up is higher, agents seem to weight less volatility in their welfare (the first channel mentioned above).

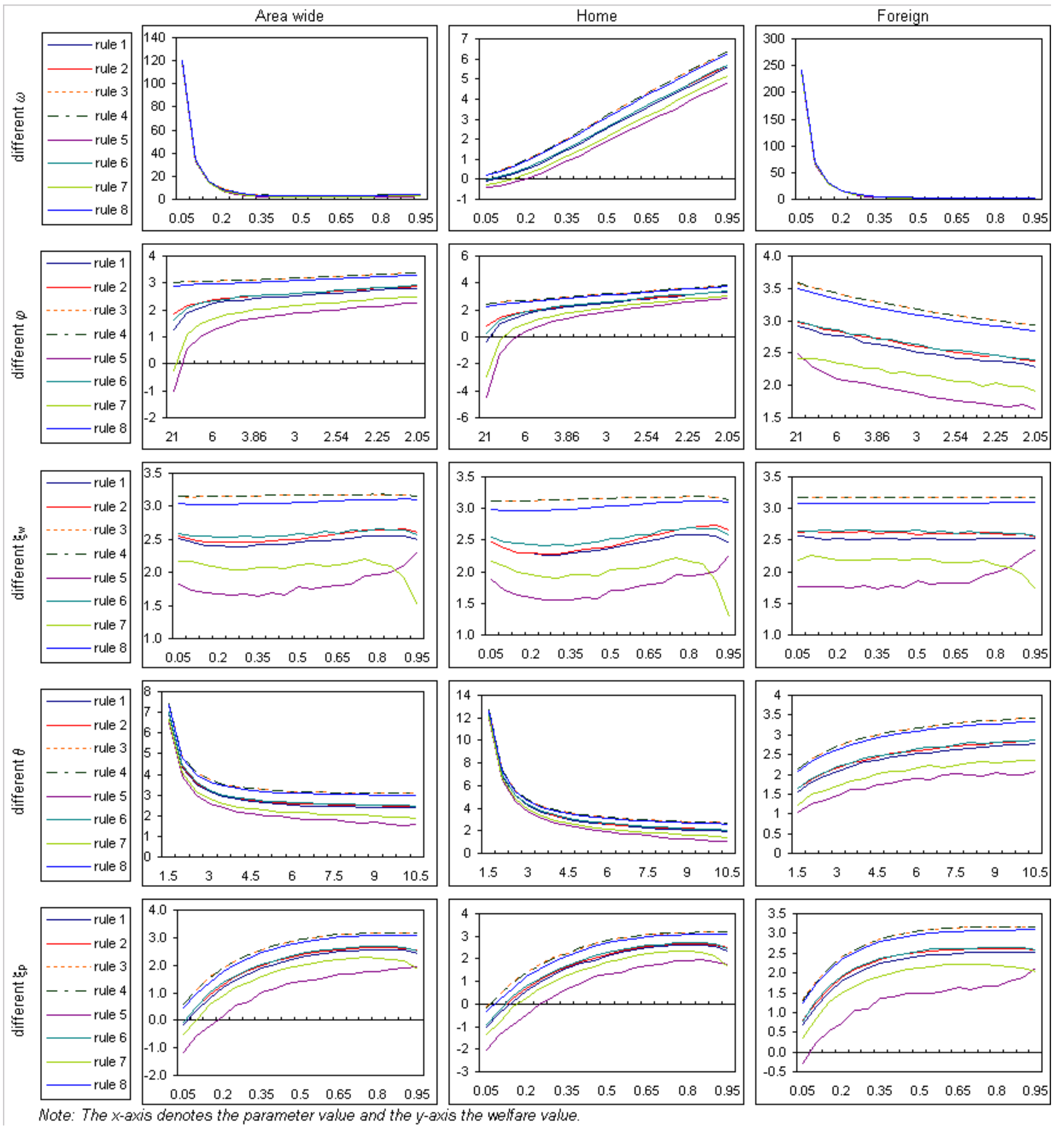
The results regarding the effects of labour and goods markets rigidity and efficiency are somehow surprising. One would expect that more flexible and more competitive economies lead to larger welfare gains for the society. The existence of heterogeneity in a currency union can justify these results. The size and persistence of shocks, as well as the fact that the central bank follows a non-optimal rule that does not respond to countries' specificities can also be considered justifiable factors.

Different degrees of home bias (ω) impact welfare only through the impact on the steady-state level, given that, as observed in section 4.2.2, do not impact in the economic variables' volatility. The impact in both countries is the opposite: the welfare in the home country increases as the home bias increases, while in the foreign economy welfare gains are extremely high for very low levels of the home bias in the domestic economy. When households dislike very much their country firms' produced goods and prefer to consume almost exclusively imported goods, their welfare is very low, as output is also low. The foreign country's firms face a large demand for their goods, which leads to a very high welfare. The foreign country welfare dominates the area wide welfare pattern.

Finally, the comparison of the results obtained by applying different policy rules suggests that simple rules, such as the Taylor rule, provide the best result regarding welfare, which is consistent with the volatility analysis (see section 4.2.2). Introducing a smoothing component and the differential component reduces welfare in both countries. Regarding the weight that the central bank puts on inflation and the output gap, we also observe that welfare is higher when the weight on the output gap or on inflation is higher. It is also worth mentioning that considering a very low weight on the inflation target leads to the worst result in terms of welfare. In other words, when the central bank follows a rule that provides a faster and stronger stabilization after shocks, this leads to the lower welfare losses.

These results are consistent with the volatility analysis presented in the previous section. Indeed, we concluded before that, when all shocks are in place, simpler rules, without interest rate smoothing, result in lower volatility. Additionally, it was also observed that when the central bank does not give much importance to inflation stabilization the main economic variables fluctuate more.

Figure 9: Welfare analysis considering different policy rules and different parameter values.



Note: The x-axis denotes the parameter value and the y-axis the welfare value.

The importance of the nominal rigidity and country heterogeneity From figure 9 we obtained some surprising results, as higher labour and goods markets efficiency and flexibility in the home country reduces welfare. One would expect that the more flexible and efficient the economies are, the better the welfare would be.

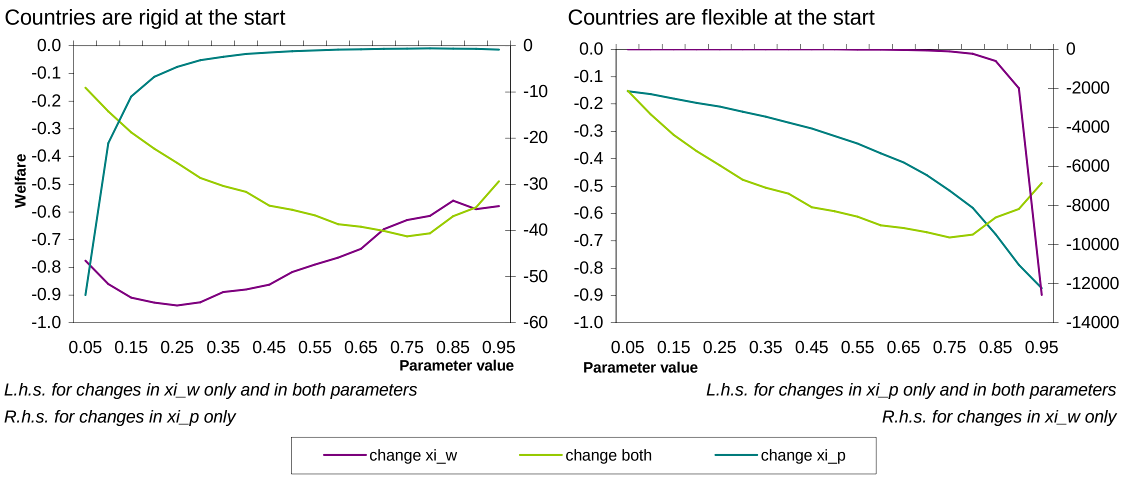
However, our analysis suggests that in a monetary union, where the common central bank responds only to the aggregate variables, what matters most for welfare is the overall rigidity in the economy and the similarities between countries[18].

Figure 10 shows the results of the simulations for the welfare log-deviations from the steady-state of both countries and the aggregate. Countries remain equal at all times, which means that rigidity parameters change at the same time in both countries. When both countries are flexible, regarding wages and prices, welfare losses are at a minimum value. Therefore, as it would be expected, nominal flexibility is indeed the best situation for both countries. As wage and price rigidity increase, welfare losses increase. If countries start from a flexible situation and increase their rigidity, welfare diminishes, and the losses are stronger and more pronounced when only wage rigidity increases. On the other hand, when countries start from the baseline scenario (sticky wages and prices), welfare does not change significantly when wages get more flexible, but it decreases strongly when price setting is more flexible. Then, from this analysis, one can conclude that price flexibility without wage flexibility is not desirable from a welfare viewpoint, as price inflation is more important for the welfare than wage inflation.

Besides nominal rigidity, it is more important for societies' welfare that the economies are similar. Indeed, from figure 11 we can observe that welfare losses are minimized when there is more homogeneity in the currency union. When the foreign country remains unchanged from the baseline scenario, increasing only price flexibility in the home economy

[18] The results and conclusions presented in this section are based on simulations with the central bank following rule 1 and with the remaining parameters equal to the baseline scenario. We have also compared different policy rules, but the conclusions regarding rules hierarchy do not change.

Figure 10: Area wide welfare log-deviations from the steady-state for different degrees of nominal rigidity, maintaining countries equal at all times.

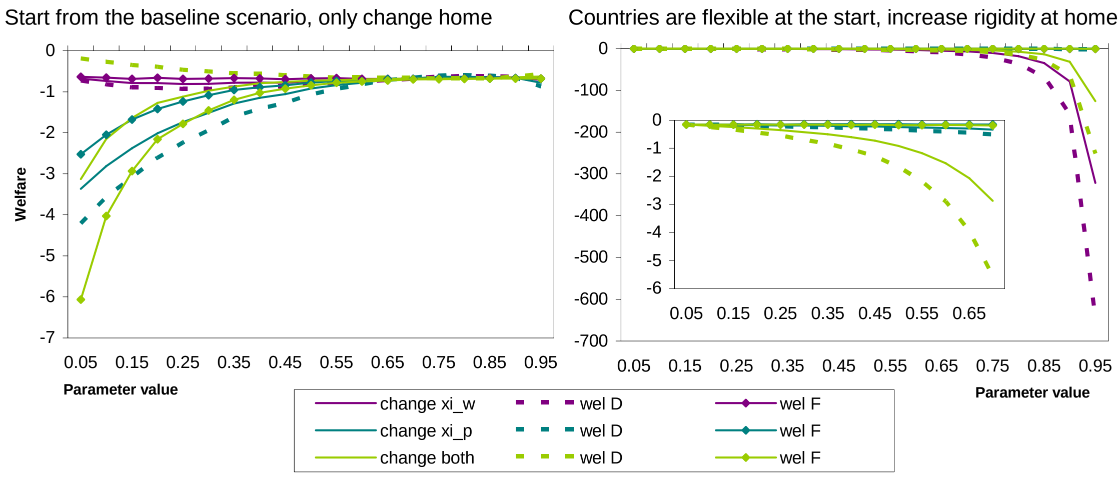


increases welfare losses in both countries and, curiously, these losses are stronger in the home country. Increasing only wage flexibility domestically does not significantly change welfare in both countries. But increasing overall flexibility in the home country improves welfare domestically but in the foreign economy it gets much worse.

On the other hand, when the foreign economy is flexible and the home economy increases its rigidity, the foreign welfare remains broadly unchanged. However, in the home economy welfare is worsened by the increased price and wage rigidity, with wage rigidity worsening more the welfare.

All in all, we find out that if countries have rigidity in their price and wage setting, in a currency union where the central bank responds to the aggregate and does not take into account national differences, then it is preferable to increase flexibility in both countries and in both wages and prices. If one country attempts to increase flexibility of its wages or prices without cooperating with the other country, the overall welfare decreases. As labour markets are specific to each country, making only wages more flexible does not significantly change welfare. However, increasing price setting flexibility while wages are

Figure 11: Welfare analysis for different degrees of nominal rigidity of the home economy, while the foreign remains unchanged.



sluggish in labour markets without mobility worsens significantly the area wide welfare, given that more adjustments are made through prices and price inflation weights more in the welfare than wage inflation.[19]

Additionally, it should be taken into consideration that these conclusions are taken assuming that the central bank does not change the rule when the parameters change, and that the results are dependent of the shocks considered. Adão et al. (2003) shows that in a currency union with only one friction and in face of common shocks, the optimal monetary policy is able to replicate the full flexibility allocation.

In addition, the central bank does not follow an optimal rule. However, simple rules seem to be a good proxy to optimal rules (Galí, 2002) and are better understood by economic agents. One could also consider that the central bank takes into account the economies' specificity, which could lead to a better result regarding welfare. For instance,

[19] In case we approximate the persistence and size of shocks to the estimates found by Smets and Wouters (2003), the conclusions do not differ regarding the welfare sensitivity to changes in the parameters. Nonetheless, it is worth mentioning that we reach different results regarding policy rules hierarchy, as there is a greater proximity among rules impact on welfare.

Benigno (2004) defends that the optimal monetary policy in a currency union where there are differences in price rigidity among countries can be approximated by an inflation targeting policy rule which gives a higher weight to inflation in the more rigid country.

5 Conclusions

The main purpose of this dissertation was to study the effects of having heterogeneous countries sharing the same currency and monetary policy. The motivation for this study comes from the acknowledgement that in the euro area there are persistent differentials, for instance regarding inflation and output growth, that can be due to a convergence process but also to structural differences between countries. Given that the Eurosystem's monetary policy responds to the euro area wide inflation, countries' idiosyncrasies do not count for policy definition.

In this context, we built a two-country DSGE model, which includes several common features of this type of model: habit formation, product and labour differentiation, monopolistic competition and price and wage rigidity. In addition, there is not labour mobility between the two countries and consumers can have different preferences for home and foreign produced goods. Countries can diverge regarding various features, but in this paper we considered that they could be different regarding the home bias, the wage and price mark-ups and the wage and price rigidity. Although each possible source of country differentiation is treated in an isolated fashion, the paper includes more sources of heterogeneity than the current literature, as far as to our knowledge.

The model is calibrated for replicating the euro area wide behaviour and is found to be able to reproduce the response of the economy to shocks relatively closer to Smets and Wouters (2003), although with lower intensity and persistence (mainly due to the lower persistence of shocks). Regarding country heterogeneity, only heterogeneity on the home bias can lead to differentials in consumer price inflation. Country heterogeneity regarding wage or price mark-ups does not lead to significantly different responses of the economies to shocks. Instead, different degrees of nominal rigidity do lead to different impacts between countries and it matters which country is hit by the shock: the response of the economy to shocks is smoother and more persistent when the shock occurs in the more sluggish country. Overall, when there is higher nominal rigidity, shocks lead to smoother and more persistent responses and results are unfavorable to the more sluggish country.

Regarding volatility and welfare assessment, one can conclude for a consistency between the two analyses. Again, the most important source of heterogeneity is nominal rigidity. There are significant changes in, namely, inflation and output volatility and welfare when we change the wage and price rigidity in both countries at the same time or only in one at a time. An increase in rigidity decreases consumer price inflation volatility, given that agents ability to reoptimize prices diminishes. Moreover, output volatility when countries have different nominal rigidity degrees is larger in the country with relatively higher rigidity. The comparison of different policy rules allows to conclude that simpler rules, without an interest rate smoothing component, permit a more stable behaviour of inflation and output. This is widely consistent with the welfare analysis. Heterogeneity regarding nominal rigidity is very important to explain welfare. In a currency union where the central bank responds to the aggregate and does not take into account national differences, then it is preferable to increase flexibility in both countries and in both wages and prices, when the starting point is the rigid scenario. There are significant welfare losses when only one country attempts to increase the flexibility of its wage or price setting or when countries attempt to make only wages or prices more flexible. These losses are more prominent when countries promote price flexibilization with rigidity on wage setting in a currency union without labour mobility.

In the sequence of this last conclusion, we can speculate whether allowing for labour mobility would change the importance of wage sluggishness in welfare. One could expect that if labour could move between countries, we would have one more mean of adjustment to shocks and countries with more rigid wages could have higher welfare losses. Therefore, allowing for labour mobility is one of the interesting possible lines of future research.

Additionally, the model could also be enriched by introducing capital, freely moving inside the area, and fiscal policy, independently set by each country. These two features would bring our model closer to the euro area's reality. The simulations made in this dissertation assume that shocks are all similar, with a relatively low persistence in comparison to some studies and with the same size. We also consider that the conclusions could be made closer to euro area's reality by using calibration values for shocks size and persistence closer to some papers estimates, such as Smets and Wouters (2003), Pytlarczyk (2005) or Jondeau and Sahuc (2006). Finally, it would also be interesting to follow a different line of research by estimating the model for one country, namely Portugal, while the foreign country of the model could be taken as remaining euro area, as Pytlarczyk (2005) has done it for Germany. This could also be useful for analyzing of the possible effects that the common monetary policy is having in Portugal.

A Appendix - The steady-state model

In this appendix, the expression of the model in the steady-state are presented. The solution is not explicitly stated given its non-linearity, but one can prove that the steady-state is unique and stable given the calibration used in the dissertation.

1. Rate of return on bonds

$$R = \frac{1}{\beta}$$

2. Real wage

$$\tilde{w}_D = \frac{\varphi}{\varphi - 1} \frac{L^{D\,\sigma_l}}{\left[C^D (1-h)\right]^{-\sigma_c}}$$

$$\tilde{w}_F = \frac{\varphi^*}{\varphi^* - 1} \frac{L^{F\,\sigma_l^*}}{\left[C^F (1-h^*)\right]^{-\sigma_c^*}}$$

3. Production function

$$\phi^D = \frac{L^D}{Y^D}$$

$$\phi^F = \frac{L^F}{Y^F}$$

4. Fixed costs

$$\phi^D = 1 + \frac{1}{\theta}$$

$$\phi^F = 1 + \frac{1}{\theta^*}$$

5. Marginal cost

$$mc^D = \frac{\theta - 1}{\theta}$$

$$mc^F = \frac{\theta^* - 1}{\theta^*}$$

6. Marginal productivity

$$1 = \frac{\varphi}{\varphi - 1} \frac{\theta}{\theta - 1} \frac{L^{D\,\sigma_l}}{\left[C^D (1-h)\right]^{-\sigma_c}}$$

$$1 = \frac{\varphi^*}{\varphi^* - 1} \frac{\theta^*}{\theta^* - 1} \frac{L^{F\,\sigma_l^*}}{\left[C^F (1-h^*)\right]^{-\sigma_c^*}}$$

7. Optimal price

$$P_D = \tilde{P}_D$$

$$P_F = \tilde{P}_F$$

8. Price parity

$$\frac{P^{cD}}{P^{cF}} = \left(\frac{P_D}{P_F}\right)^{\varpi - \varpi^*}$$

9. Terms of trade

$$T = 1$$

10. Market equilibrium

$$Y^D = \varpi C^D + \frac{1-n}{n}\varpi^* C^F$$

$$Y^F = (1-\varpi)\frac{n}{1-n}C^D + (1-\varpi^*)\, C^F$$

11. Aggregate economy

$$Y = C$$

B Appendix - Equations for the foreign economy in the log-linearized model

1. Consumption equation

$$\hat{C}_t^F = \frac{h^*}{1+h^*}\hat{C}_{t-1}^F + \frac{1}{1+h^*}E_t\hat{C}_{t+1}^F + \frac{1-h^*}{\sigma_c^*(1+h^*)}\left(\hat{\varepsilon}_t^{bF} - E_t\hat{\varepsilon}_{t+1}^{bF}\right) - \frac{1-h^*}{\sigma_c^*(1+h^*)}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}^{cF}\right)$$

2. Preference shock

$$\hat{\varepsilon}_t^{bF} = \rho_B^*\hat{\varepsilon}_{t-1}^{bF} + \xi_t^{bF}$$

3. Real wage

$$\begin{aligned}\hat{w}_{F,t} = {} & \frac{\beta}{1+\beta}E_t\hat{w}_{F,t+1} + \frac{1}{1+\beta}\hat{w}_{F,t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1}^{cF} - \frac{1+\beta\gamma_w^*}{1+\beta}\hat{\pi}_t^{cF} + \frac{\gamma_w^*}{1+\beta}\hat{\pi}_{t-1}^{cF} \\ & - \frac{1}{1+\beta}\frac{\left(1-\beta\xi_w^F\right)\left(1-\xi_w^F\right)}{(1+\varphi^*\sigma_L^*)\,\xi_w^F}\left[\hat{w}_{F,t} - \sigma_L^*\hat{L}_t^F - \frac{\sigma_c^*}{1-h^*}\left(\hat{C}_t^F - h^*\hat{C}_{t-1}^F\right) - \hat{\varepsilon}_t^{LF}\right]\end{aligned}$$

4. Labour supply shock

$$\hat{\varepsilon}_t^{LF} = \rho_L^* \hat{\varepsilon}_{t-1}^{LF} + \xi_t^{LF}$$

5. Producer price inflation

$$\hat{\pi}_{F,t} = \frac{\beta}{1+\beta\gamma_p^*} E_t \hat{\pi}_{F,t+1} + \frac{\gamma_p^*}{1+\beta\gamma_p^*} \hat{\pi}_{F,t-1} + \frac{1}{1+\beta\gamma_p^*} \frac{\left(1-\beta\xi_p^F\right)\left(1-\xi_p^F\right)}{\xi_p^F} \left(\hat{w}_{F,t} - \hat{A}_t^F\right)$$

6. Productivity shock

$$\hat{A}_t^F = \rho_a^* \hat{A}_t^F + \hat{\eta}_{a,t}^F$$

7. Consumer price inflation

$$\hat{\pi}_t^{cF} = \varpi^* \hat{\pi}_{D,t} + (1-\varpi^*)\, \hat{\pi}_{F,t}$$

8. Aggregate production function

$$\hat{Y}_t^F = \phi^F \left(\hat{A}_t^F + \hat{L}_t^F\right)$$

9. Market clearing

$$\hat{Y}_t^F = -\hat{T}_t \left[\varpi \left(1-(1-\varpi^*)\, c_y^F\right) + \varpi^* (1-\varpi^*)\, c_y^F\right] + \left(1-(1-\varpi^*)\, c_y^F\right) \hat{C}_t^D + (1-\varpi^*)\, c_y^F \hat{C}_t^F$$

C Appendix - Model dynamics with heterogeneous countries

Figure 12: I.r.f. when countries are homogeneous.

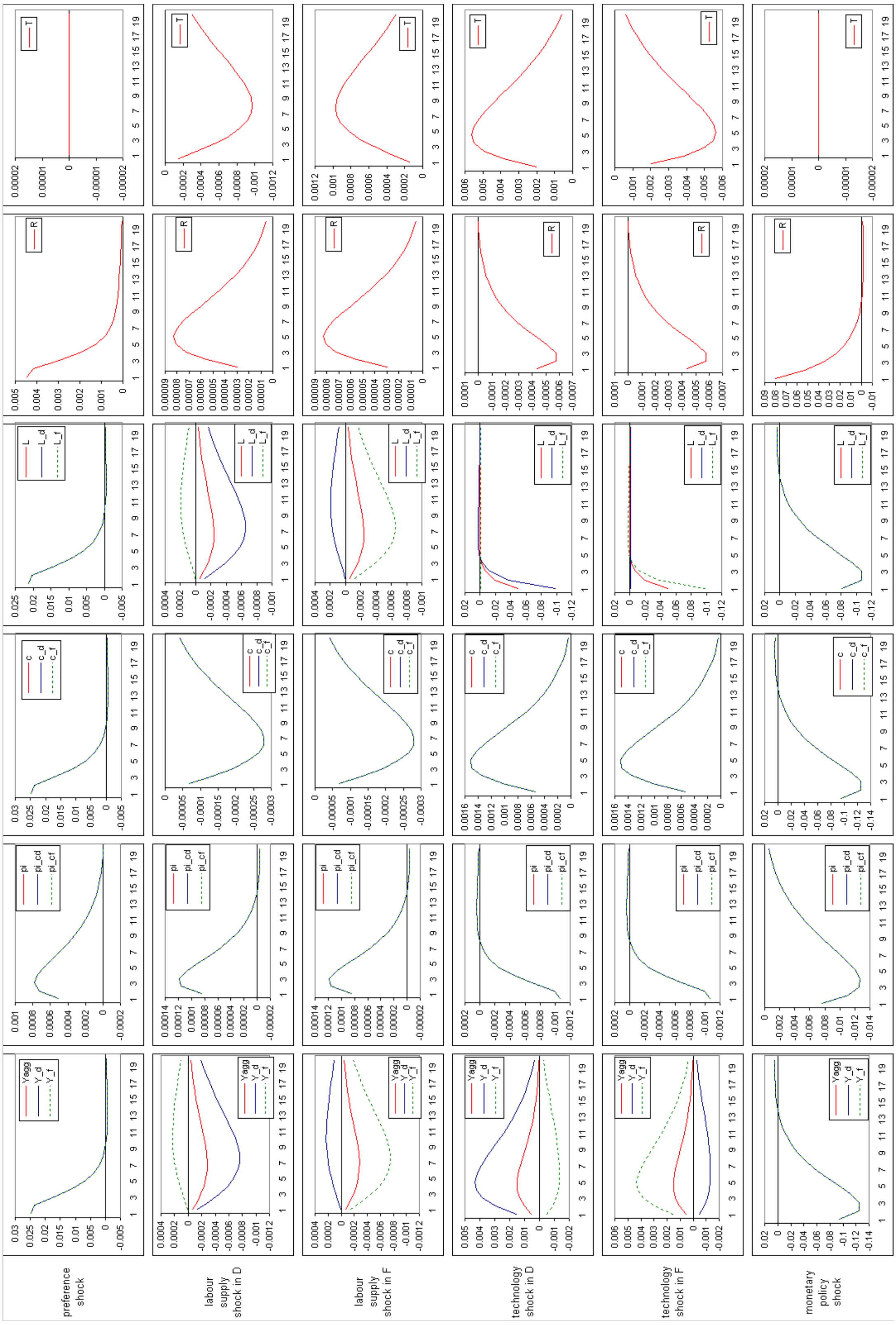


Figure 13: I.r.f. when the home country differs on the home bias (ϖ).

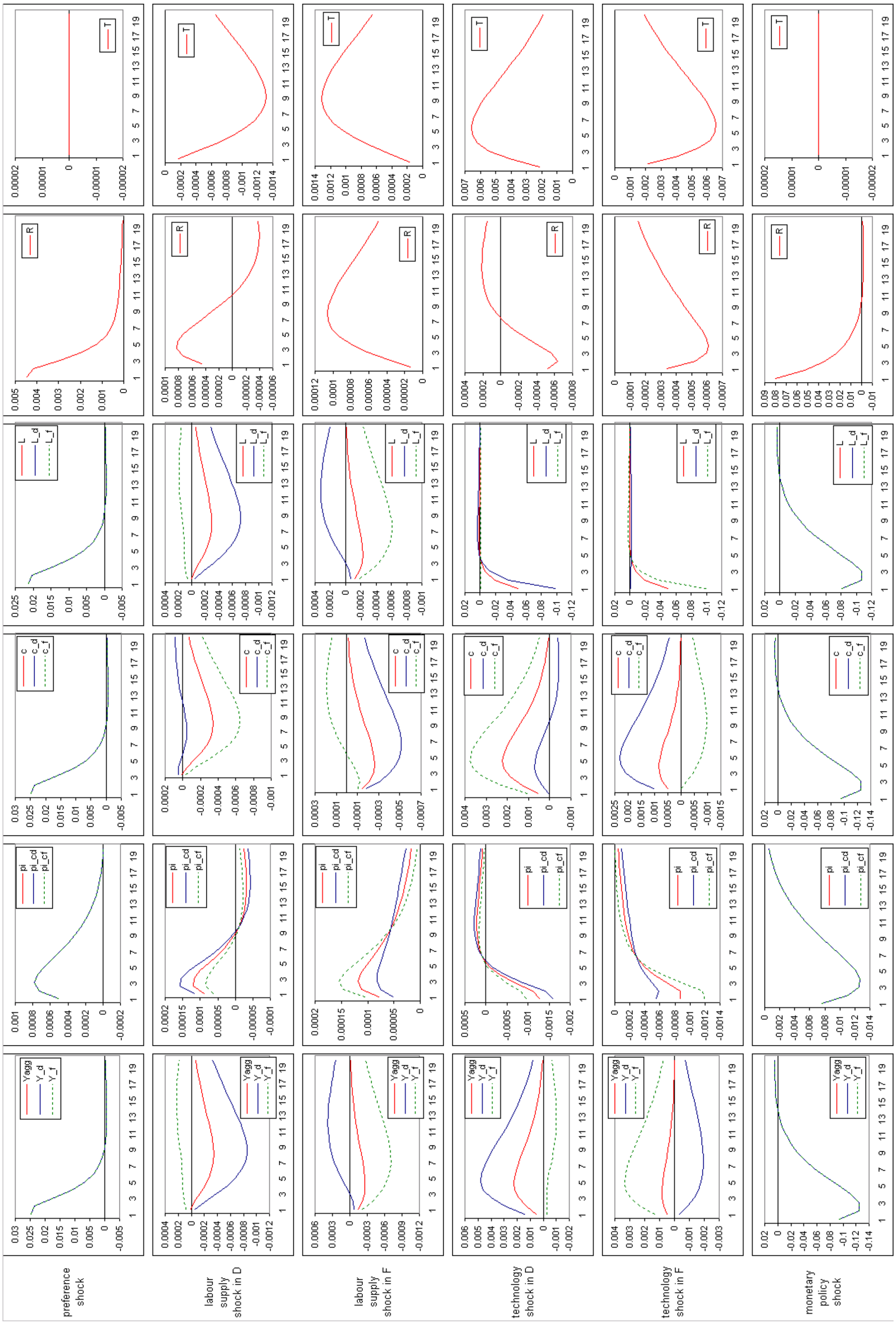


Figure 14: I.r.f. when the home country differs on the wage mark-up (φ).

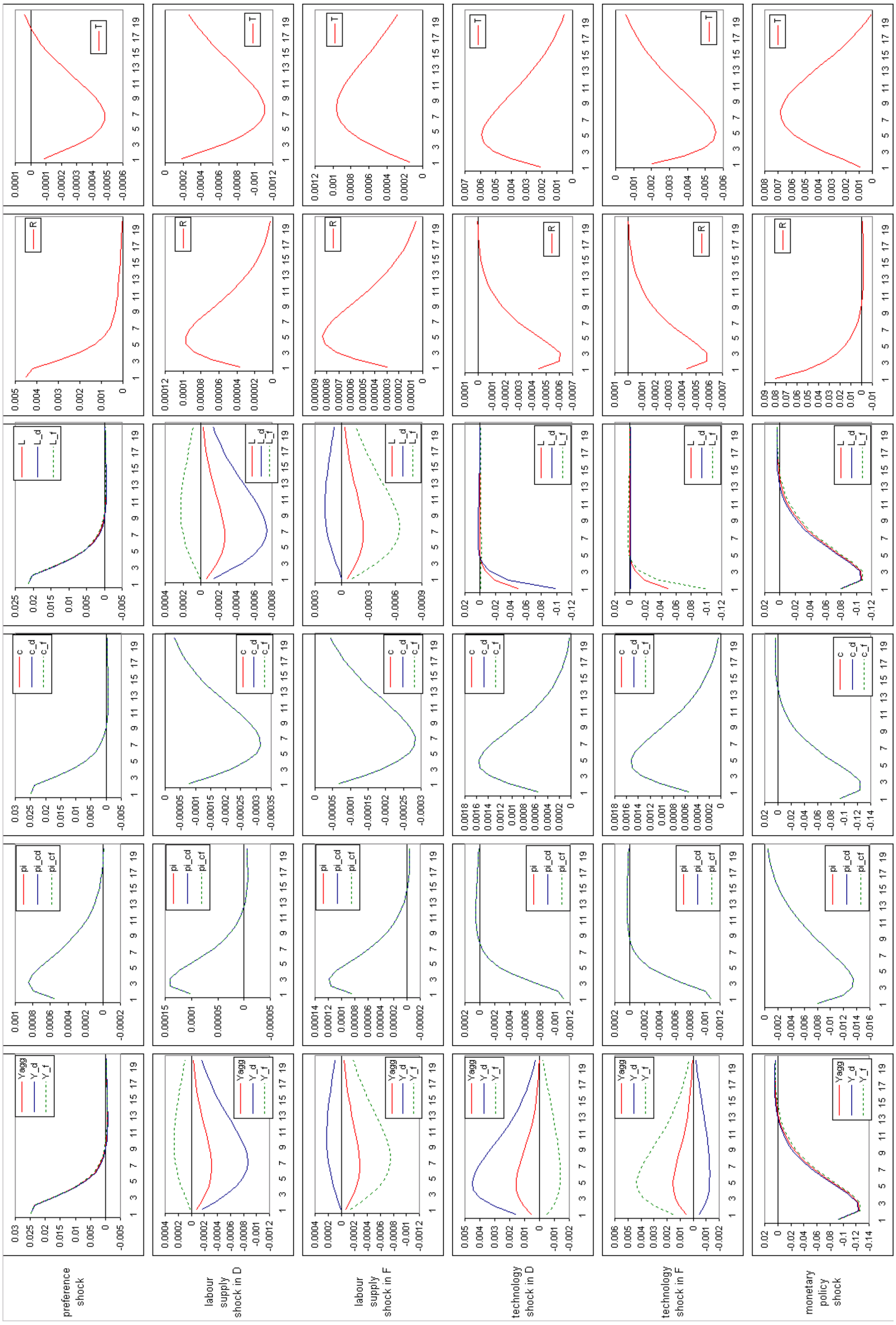


Figure 15: I.r.f. when the home country differs on the wage rigidity (ξ_w^D).

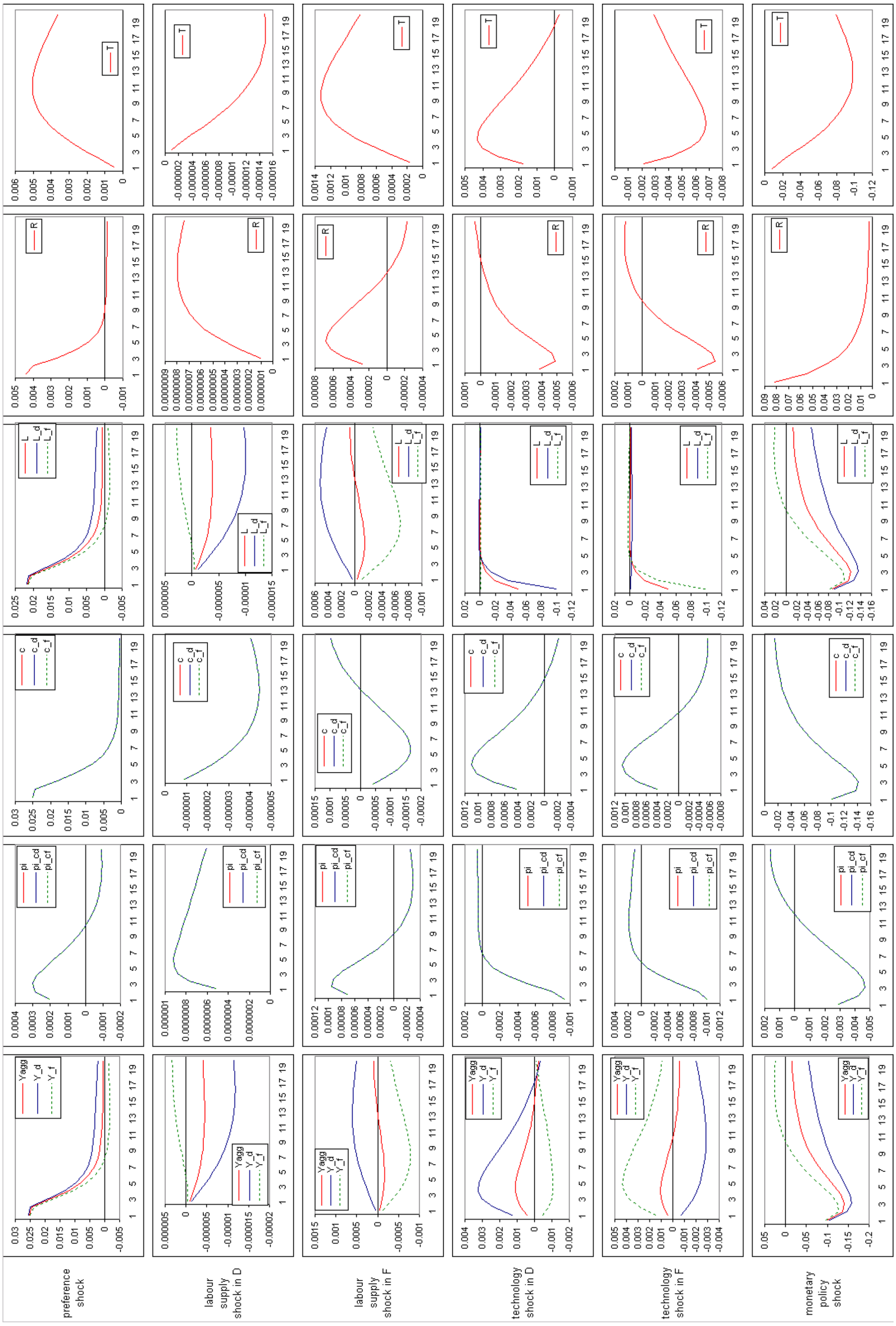


Figure 16: I.r.f. when the home country differs on the price mark-up (θ).

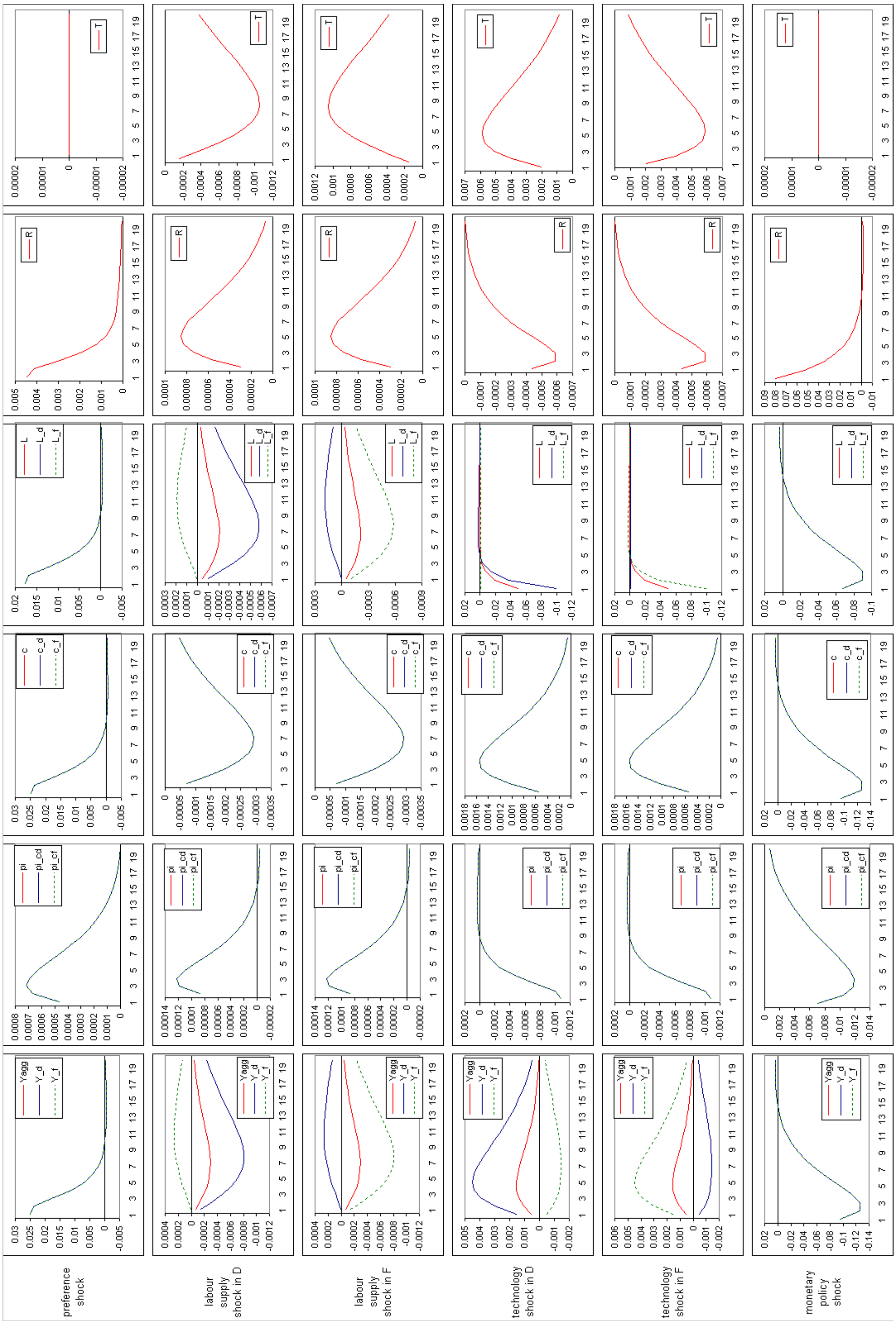
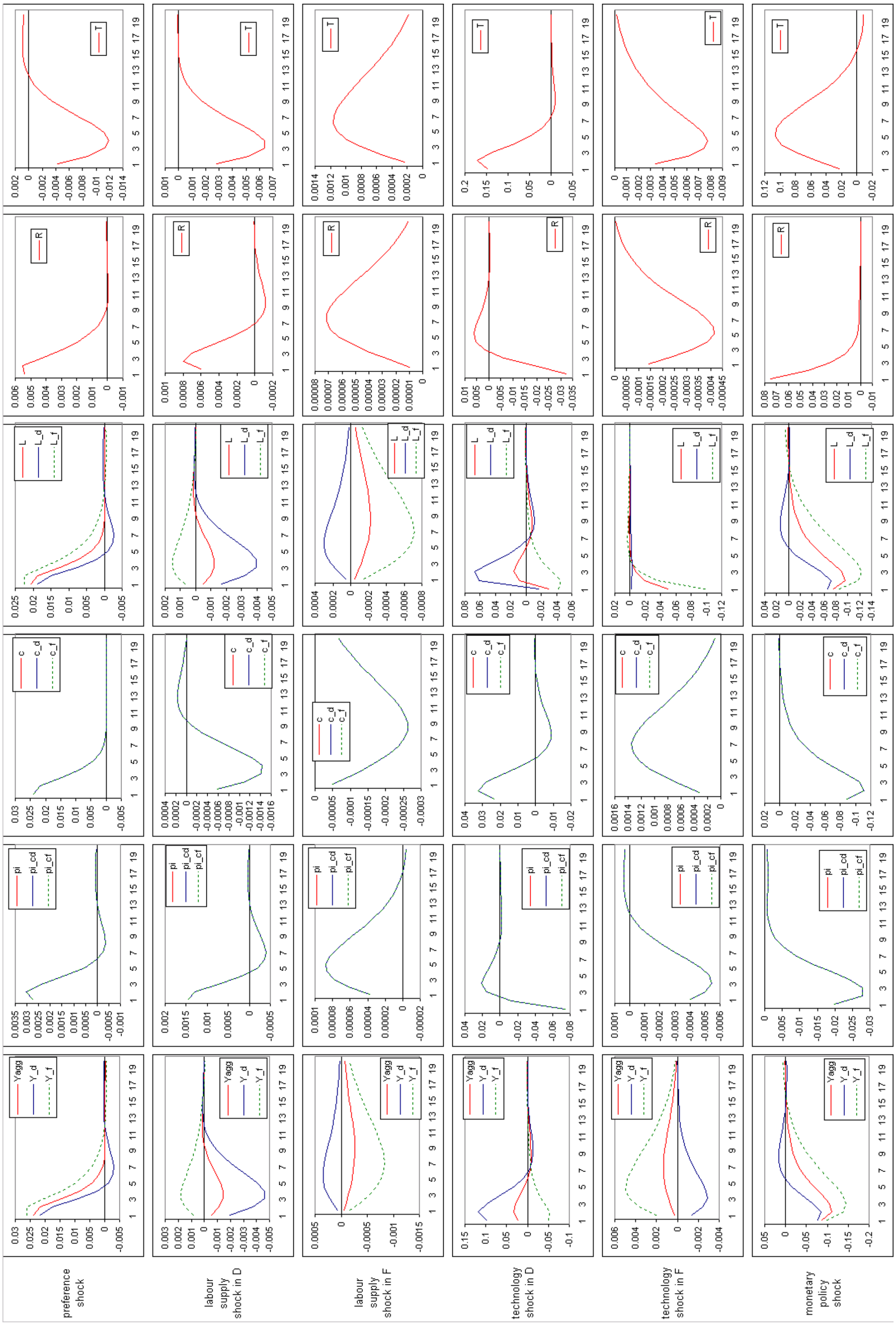


Figure 17: I.r.f. when the home country differs on the price rigidity (ξ_p^D).



D Appendix - Determination of the welfare function

The determination of the welfare expression follows the methods for the approximation of loss functions discussed in Woodford (2003) and Benigno and Woodford (2004). The average utility of the representative household at moment t is given by:

$$\mathcal{W}_t = U(C_t, H_t, \varepsilon_t^b) - \frac{1}{n}\int_0^n V(L_t\,(i)\,, \varepsilon_t^b, \varepsilon_t^L)di \tag{60}$$

where: $U(C_t, H_t, \varepsilon_t^b) = \dfrac{\varepsilon_t^b}{1-\sigma_c}\,(C_t - H_t)^{1-\sigma_c}$ is the utility from consumption and $V(L_t\,(i)\,, \varepsilon_t^b, \varepsilon_t^L) = \dfrac{\varepsilon_t^b \varepsilon_t^L}{1+\sigma_l}\,L_t\,(i)^{1+\sigma_l}$ is the disutility from working, and both are defined from the utility expression (2).

D.1 Approximation of the utility from consumption

Following Woodford (2003), we now compute a quadratic Taylor series approximation to (60). First, we get the following result for the second-order Taylor expansion of $U(C_t, H_t, \varepsilon_t^b)$ around the steady state $\bar{U} = U(C, H, \varepsilon^b)$:

$$U(C_t, H_t, \varepsilon_t^b) \approx \bar{U} + \bar{U}_C\tilde{C}_t + \bar{U}_H\tilde{H}_t + \bar{U}_{\varepsilon^b}\tilde{\varepsilon}_t^b + \tfrac{1}{2}\bar{U}_{CC}\tilde{C}_t^2 + \tfrac{1}{2}\bar{U}_{HH}\tilde{H}_t^2 + \tfrac{1}{2}\bar{U}_{\varepsilon^b\varepsilon^b}\;{\tilde{\varepsilon}_t^b}{}^2 +$$
$$+\bar{U}_{CH}\tilde{C}_t\tilde{H}_t + \bar{U}_{C\varepsilon^b}\tilde{C}_t\tilde{\varepsilon}_t^b + \bar{U}_{H\varepsilon^b}\tilde{H}_t\tilde{\varepsilon}_t^b + \mathcal{O}\left(\|\zeta\|^3\right)$$

where a variable with a til means its deviation from the steady state ($\tilde{x}_t = x_t - x$) and $\mathcal{O}\left(\|\zeta\|^3\right)$ denotes the order of the residual and $\|\zeta\|$ is a bound on the amplitude of exogenous disturbances. Substituting in the above expression $\tilde{x}_t$ for $\hat{x}_t$ using a Taylor series expansion $\dfrac{x_t}{x} = 1 + \hat{x}_t + \frac{1}{2}\hat{x}_t^2 + \mathcal{O}\left(\|\zeta\|^3\right)$ (where $\hat{x}_t = \ln x_t - \ln x$), it yields:

$$U(C_t, H_t, \varepsilon_t^b) \approx \bar{U} + \bar{U}_C C\left(\hat{C}_t + \tfrac{1}{2}\hat{C}_t^2\right) + \bar{U}_H H\left(\hat{H}_t + \tfrac{1}{2}\hat{H}_t^2\right) + \bar{U}_{\varepsilon^b}\varepsilon^b\left(\hat{\varepsilon}_t^b + \tfrac{1}{2}\;{\hat{\varepsilon}_t^b}{}^2\right) +$$
$$+\tfrac{1}{2}\bar{U}_{CC}C^2\hat{C}_t^2 + \tfrac{1}{2}\bar{U}_{HH}H^2\hat{H}_t^2 + \tfrac{1}{2}\bar{U}_{\varepsilon^b\varepsilon^b}\;{\varepsilon^b}^2\;{\hat{\varepsilon}_t^b}{}^2 + \bar{U}_{CH}CH\hat{C}_t\hat{H}_t +$$
$$+\bar{U}_{C\varepsilon^b}C\varepsilon^b\hat{C}_t\hat{\varepsilon}_t^b + \bar{U}_{H\varepsilon^b}H\varepsilon^b\hat{H}_t\hat{\varepsilon}_t^b + \mathcal{O}\left(\|\zeta\|^3\right)$$

Derivating, we have

$\bar{U}_C = \varepsilon^b\,(C-H)^{-\sigma_c}$

$\bar{U}_H = -\varepsilon^b\,(C-H)^{-\sigma_c} = -\bar{U}_C$

$$\bar{U}_{\varepsilon^b} = \frac{1}{1-\sigma_c}\left(C-H\right)^{1-\sigma_c} = \frac{(C-H)}{(1-\sigma_c)\,\varepsilon^b}\bar{U}_C$$
$$\bar{U}_{CC} = -\sigma_c\varepsilon^b\left(C-H\right)^{-\sigma_c-1} = \frac{-\sigma_c}{(C-H)}\bar{U}_C$$
$$\bar{U}_{HH} = -\sigma_c\varepsilon^b\left(C-H\right)^{-\sigma_c-1} = \frac{-\sigma_c}{(C-H)}\bar{U}_C$$
$$\bar{U}_{\varepsilon^b\varepsilon^b} = 0$$
$$\bar{U}_{CH} = \sigma_c\varepsilon^b\left(C-H\right)^{-\sigma_c-1} = \frac{\sigma_c}{(C-H)}\bar{U}_C$$
$$\bar{U}_{C\varepsilon^b} = \left(C-H\right)^{-\sigma_c} = \frac{1}{\varepsilon^b}\bar{U}_C$$
$$\bar{U}_{H\varepsilon^b} = -\left(C-H\right)^{-\sigma_c} = -\frac{1}{\varepsilon^b}\bar{U}_C$$

Replacing $H_t = hC_{t-1}$ yields

$$U(C_t, H_t, \varepsilon_t^b) \approx \bar{U} + \bar{U}_C C \begin{bmatrix} \left(\hat{C}_t - h\hat{C}_{t-1}\right) + \frac{1}{2}\left(\hat{C}_t^2 - h^2\hat{C}_{t-1}^2\right) + \frac{1-h}{2\left(1-\sigma_c\right)}\,\hat{\varepsilon}_t^{b\,2} - \\ -\frac{\sigma_c}{2\left(1-h\right)}\left(\hat{C}_t - h^2\hat{C}_{t-1}\right)^2 + \hat{\varepsilon}_t^b\left(\hat{C}_t - h^2\hat{C}_{t-1}\right) \end{bmatrix} + \\ +\mathcal{O}\left(\|\zeta\|^3\right)$$

D.2 Approximation of the disutility from working

If we integrate the disutility from working over the population of the economy, we get

$$\frac{1}{n}\int_0^n V(L_t\left(i\right), \varepsilon_t^b, \varepsilon_t^L)di = \frac{1}{n}\int_0^n \frac{\varepsilon_t^b\varepsilon_t^L}{1+\sigma_l}\,L_t\left(i\right)^{1+\sigma_L}\,di = \frac{1}{n}\frac{n\varepsilon_t^b\varepsilon_t^L}{1+\sigma_l}\int_0^n\left[\left(\frac{W_t\left(i\right)}{W_t}\right)^{-\varphi}\frac{L_t}{n}\right]^{1+\sigma_L} di \Leftrightarrow$$
$$\frac{1}{n}\int_0^n V(L_t\left(i\right), \varepsilon_t^b, \varepsilon_t^L)di = \frac{\varepsilon_t^b\varepsilon_t^L}{1+\sigma_l}\left(\frac{L_t}{n}\right)^{1+\sigma_L}\int_0^n\left(\frac{W_t\left(i\right)}{W_t}\right)^{-\varphi(1+\sigma_L)} di \Leftrightarrow$$
$$\frac{1}{n}\int_0^n V(L_t\left(i\right), \varepsilon_t^b, \varepsilon_t^L)di = \frac{\varepsilon_t^b\varepsilon_t^L}{1+\sigma_l}\left(\frac{L_t}{n}\right)^{1+\sigma_L}\Delta_{w,t} = V(L_t, \varepsilon_t^b, \varepsilon_t^L)\Delta_{w,t}$$

given equation (12). The variable $\Delta_{w,t} = \int_0^n\left(\frac{W_t\left(i\right)}{W_t}\right)^{-\varphi(1+\sigma_L)} di$ denotes the measure of wage dispersion at date t (see Benigno and Woodford, 2004). The approximation of $V(L_t, \varepsilon_t^b, \varepsilon_t^L)\Delta_{w,t}$ follows Benigno and Woodford (2004):

$$V(L_t, \varepsilon_t^b, \varepsilon_t^L)\Delta_{w,t} = \bar{V} + \bar{V}\left(\Delta_{w,t}-1\right) + \bar{V}_L\tilde{L}_t + \bar{V}_L\left(\Delta_{w,t}-1\right)\tilde{L}_t + \bar{V}_{\varepsilon^b}\tilde{\varepsilon}_t^b\left(\Delta_{w,t}-1\right) +$$
$$+\bar{V}_{\varepsilon^L}\tilde{\varepsilon}_t^L\left(\Delta_{w,t}-1\right) + \tfrac{1}{2}\bar{V}_{LL}\tilde{L}_t + \tfrac{1}{2}\bar{V}_{\varepsilon^b\varepsilon^b}\,\tilde{\varepsilon}_t^{b\,2} + \tfrac{1}{2}\bar{V}_{\varepsilon^L\varepsilon^L}\,\tilde{\varepsilon}_t^{L\,2} +$$
$$+\bar{V}_{L\varepsilon^b}\tilde{L}_t\tilde{\varepsilon}_t^b + \bar{V}_{L\varepsilon^L}\tilde{L}_t\tilde{\varepsilon}_t^L + \mathcal{O}\left(\|\zeta\|^3\right)$$

Derivating $V\left(.\right)$, we have:

$$\bar{V}_L = \frac{\varepsilon^b\varepsilon^L}{n}\left(\frac{L}{n}\right)^{\sigma_L}\Delta_w = \frac{\varepsilon^b\varepsilon^L}{n}\left(\frac{L}{n}\right)^{\sigma_L}$$
$$\bar{V}_{\varepsilon^b} = \frac{\varepsilon^L}{1+\sigma_L}\left(\frac{L}{n}\right)^{1+\sigma_l}\Delta_w = \frac{L}{\varepsilon^b\left(1+\sigma_L\right)}\bar{V}_L$$

$$\bar{V}_{\varepsilon^L} = \frac{\varepsilon^b}{1+\sigma_L}\left(\frac{L}{n}\right)^{1+\sigma_l}\Delta_w = \frac{L}{\varepsilon^L(1+\sigma_L)}\bar{V}_L$$
$$\bar{V}_{LL} = \frac{\sigma_L\varepsilon^b\varepsilon^L}{n^2}\left(\frac{L}{n}\right)^{\sigma_l-1}\Delta_w = \frac{\sigma_L}{L}\bar{V}_L$$
$$\bar{V}_{\varepsilon^b\varepsilon^b} = 0$$
$$\bar{V}_{\varepsilon^L\varepsilon^L} = 0$$
$$\bar{V}_{L\varepsilon^b} = \frac{\varepsilon^L}{n}\left(\frac{L}{n}\right)^{\sigma_L}\Delta_w = \frac{1}{\varepsilon^b}\bar{V}_L$$
$$\bar{V}_{L\varepsilon^L} = \frac{\varepsilon^b}{n}\left(\frac{L}{n}\right)^{\sigma_L}\Delta_w = \frac{1}{\varepsilon^L}\bar{V}_L$$

given that in the steady state there is no price dispersion, so that $\Delta_w = 1$.

Replacing the derivatives in the equation $V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t}$ and taking into account that the deviations from the steady state of the variables in levels can be approximated by a second-order expansion in terms of the deviations from the steady state of the variables in logs, i.e., $\tilde{x}_t = \hat{x}_t + \frac{1}{2}\hat{x}_t^2$, then we have

$$\begin{aligned}
V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t} = {} & \bar{V} + \bar{V}\left(\hat{\Delta}_{w,t} + \tfrac{1}{2}\hat{\Delta}_{w,t}^2\right) + \bar{V}_L L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right) + \bar{V}_L\left(\hat{\Delta}_{w,t} + \tfrac{1}{2}\hat{\Delta}_{w,t}^2\right)\times \\
& \times L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right) + \frac{L}{\varepsilon^b(1+\sigma_L)}\bar{V}_L\varepsilon^b\left(\hat{\varepsilon}_t^b + \tfrac{1}{2}\,{\hat{\varepsilon}_t^b}^2\right)\left(\hat{\Delta}_{w,t} + \tfrac{1}{2}\hat{\Delta}_{w,t}^2\right) + \\
& + \frac{L}{\varepsilon^L(1+\sigma_L)}\bar{V}_L\varepsilon^L\left(\hat{\varepsilon}_t^L + \tfrac{1}{2}\,{\hat{\varepsilon}_t^L}^2\right)\left(\hat{\Delta}_{w,t} + \tfrac{1}{2}\hat{\Delta}_{w,t}^2\right) + \tfrac{1}{2}\frac{\sigma_L}{L}\bar{V}_L\left[L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right)\right]^2 + \\
& + \frac{1}{\varepsilon^b}\bar{V}_L L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right)\varepsilon^b\left(\hat{\varepsilon}_t^b + \tfrac{1}{2}\,{\hat{\varepsilon}_t^b}^2\right) + \frac{1}{\varepsilon^L}\bar{V}_L L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right)\varepsilon^L\left(\hat{\varepsilon}_t^L + \tfrac{1}{2}\,{\hat{\varepsilon}_t^L}^2\right) + \\
& + \mathcal{O}\left(\|\zeta\|^3\right) \Leftrightarrow
\end{aligned}$$
$$\begin{aligned}
V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t} = {} & \bar{V} + \bar{V}\hat{\Delta}_{w,t} + \bar{V}_L L\left(\hat{L}_t + \tfrac{1}{2}\hat{L}_t^2\right) + \bar{V}_L L\hat{\Delta}_{w,t}\hat{L}_t + \frac{\bar{V}_L L}{1+\sigma_L}\hat{\varepsilon}_t^b\hat{\Delta}_{w,t} + \\
& + \frac{\bar{V}_L L}{1+\sigma_L}\hat{\varepsilon}_t^L\hat{\Delta}_{w,t} + \frac{\sigma_L\bar{V}_L L}{2}\hat{L}_t^2 + \bar{V}_L L\hat{L}_t\hat{\varepsilon}_t^b + \bar{V}_L L\hat{L}_t\hat{\varepsilon}_t^L + \mathcal{O}\left(\|\zeta\|^3\right) \Leftrightarrow
\end{aligned}$$
$$\begin{aligned}
V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t} = {} & \bar{V}\left(1+\hat{\Delta}_{w,t}\right) + \\
& + \bar{V}_L L\left[\hat{L}_t + \frac{(1+\sigma_L)\hat{L}_t^2}{2} + \hat{\Delta}_{w,t}\hat{L}_t + \hat{L}_t\left(\hat{\varepsilon}_t^b + \hat{\varepsilon}_t^L\right) + \frac{\hat{\Delta}_{w,t}\left(\hat{\varepsilon}_t^b + \hat{\varepsilon}_t^L\right)}{1+\sigma_L}\right] + \mathcal{O}\left(\|\zeta\|^3\right)
\end{aligned}$$

Surpressing the terms of superior order, we reach

$$V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t} = \bar{V}\left(1+\hat{\Delta}_{w,t}\right) + \bar{V}_L L\left[\hat{L}_t + \frac{(1+\sigma_L)\hat{L}_t^2}{2} + \hat{L}_t\left(\hat{\varepsilon}_t^b + \hat{\varepsilon}_t^L\right)\right] + \mathcal{O}\left(\|\zeta\|^3\right) \tag{61}$$

The result above was obtained taking into consideration that $\hat{\Delta}_{w,t}$ is of second-order.

D.2.1 Approximation of the wage dispersion measure

In order to determine the law of motion of the wage dispersion measure, we can expand $\Delta_{w,t}$ as a sum (where W_t^* is the optimal wage at date t):

$$\Delta_{w,t} = (1-\xi_w)\left(\frac{W_t^*}{W_t}\right)^{-\varphi(1+\sigma_L)} + \xi_w (1-\xi_w)\left(\frac{W_{t-1}^*\pi_{t-1}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)} +$$
$$+\xi_w^2 (1-\xi_w)\left(\frac{W_{t-2}^*\pi_{t-2}^{\gamma_w}\pi_{t-1}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)} +\xi_w^3 (1-\xi_w)\left(\frac{W_{t-3}^*\pi_{t-3}^{\gamma_w}\pi_{t-2}^{\gamma_w}\pi_{t-1}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)} +$$

... $\Leftrightarrow$

$$\Delta_{w,t} = (1-\xi_w)\left(\frac{W_t^*}{W_t}\right)^{-\varphi(1+\sigma_L)} + \sum_{j=1}^{\infty}\xi_w^j (1-\xi_w)\left(\frac{W_{t-j}^*\prod_{i=1}^{j}\pi_{t-i}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)} \tag{62}$$

One period earlier we have:

$$\Delta_{w,t-1} = (1-\xi_w)\left(\frac{W_{t-1}^*}{W_{t-1}}\right)^{-\varphi(1+\sigma_L)} + \sum_{j=1}^{\infty}\xi_w^j (1-\xi_w)\left(\frac{W_{t-1-j}^*\prod_{i=1}^{j}\pi_{t-1-i}^{\gamma_w}}{W_{t-1}}\right)^{-\varphi(1+\sigma_L)} \Leftrightarrow$$

$$\Delta_{w,t-1} = (1-\xi_w)\left(\frac{W_{t-1}^*}{W_t}\right)^{-\varphi(1+\sigma_L)}\left(\frac{W_t}{W_{t-1}}\right)^{-\varphi(1+\sigma_L)} +$$
$$+\sum_{j=1}^{\infty}\xi_w^j (1-\xi_w)\left(\frac{W_{t-1-j}^*\prod_{i=1}^{j}\pi_{t-1-i}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)}\left(\frac{W_t}{W_{t-1}}\right)^{-\varphi(1+\sigma_L)} \Leftrightarrow$$

$$\Delta_{w,t-1} = (1-\xi_w)\left(\frac{W_{t-1}^*}{W_t}\right)^{-\varphi(1+\sigma_L)}(\pi_{w,t})^{-\varphi(1+\sigma_L)} +$$
$$+\sum_{j=1}^{\infty}\xi_w^j (1-\xi_w)\left(\frac{W_{t-1-j}^*\prod_{i=1}^{j}\pi_{t-1-i}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)}(\pi_{w,t})^{-\varphi(1+\sigma_L)}$$

where $\pi_{w,t}$ is the rate of change of wages $\pi_{w,t} = \dfrac{W_t}{W_{t-1}}$. Now, multiply $\Delta_{w,t-1}$ by ξ_w, $(\pi_{w,t})^{\varphi(1+\sigma_L)}$ and $\pi_{t-1}^{-\gamma_w\varphi(1+\sigma_L)}$:

$$\xi_w(\pi_{w,t})^{\varphi(1+\sigma_L)}\pi_{t-1}^{-\gamma_w\varphi(1+\sigma_L)}\Delta_{w,t-1} = \xi_w(1-\xi_w)\left(\frac{W_{t-1}^*}{W_t}\pi_{t-1}^{\gamma_w}\right)^{-\varphi(1+\sigma_L)} +$$
$$+\xi_w\sum_{j=1}^{\infty}\xi_w^j (1-\xi_w)\left(\frac{W_{t-1-j}^*\prod_{i=1}^{j}\pi_{t-i}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)} \Leftrightarrow$$

$$\xi_w\left(\pi_{w,t}\right)^{\varphi(1+\sigma_L)}\pi_{t-1}^{-\gamma_w\varphi(1+\sigma_L)}\Delta_{w,t-1}=\sum_{j=1}^{\infty}\xi_w^j\left(1-\xi_w\right)\left(\frac{W_{t-j}^*\prod_{i=1}^{j}\pi_{t-i}^{\gamma_w}}{W_t}\right)^{-\varphi(1+\sigma_L)}$$

The right-hand side of the above expression is equal to the second term of the right-hand side of equation (62). Therefore, we get:

$$\Delta_{w,t}=\left(1-\xi_w\right)\left(\frac{W_t^*}{W_t}\right)^{-\varphi(1+\sigma_L)}+\xi_w\left(\pi_{w,t}\right)^{\varphi(1+\sigma_L)}\pi_{t-1}^{-\gamma_w\varphi(1+\sigma_L)}\Delta_{w,t-1} \tag{63}$$

From equation (14), we can define the ratio $\frac{W_t^*}{W_t}$ and then replace in equation (63):

$W_t^{1-\varphi}=\xi_w\left[W_{t-1}\pi_{t-1}^{\gamma_w}\right]^{1-\varphi}+\left(1-\xi_w\right)\left(W_t^*\right)^{1-\varphi}\Leftrightarrow$

$1=\xi_w\left[\frac{W_{t-1}}{W_t}\pi_{t-1}^{\gamma_w}\right]^{1-\varphi}+\left(1-\xi_w\right)\left(\frac{W_t^*}{W_t}\right)^{1-\varphi}\Leftrightarrow$

$\left(1-\xi_w\right)\left(\frac{W_t^*}{W_t}\right)^{1-\varphi}=1-\xi_w\pi_{w,t}^{\varphi-1}\pi_{t-1}^{\gamma_w(1-\varphi)}\Leftrightarrow$

$\frac{W_t^*}{W_t}=\left[\frac{1-\xi_w\pi_{w,t}^{\varphi-1}\pi_{t-1}^{\gamma_w(1-\varphi)}}{1-\xi_w}\right]^{\frac{1}{1-\varphi}}$

And finally we get

$$\Delta_{w,t}=\xi_w\left(\pi_{w,t}\right)^{\varphi(1+\sigma_L)}\pi_{t-1}^{-\gamma_w\varphi(1+\sigma_L)}\Delta_{w,t-1}+\left(1-\xi_w\right)\left[\frac{1-\xi_w\pi_{w,t}^{\varphi-1}\pi_{t-1}^{\gamma_w(1-\varphi)}}{1-\xi_w}\right]^{\frac{-\varphi(1+\sigma_L)}{1-\varphi}} \tag{64}$$

We now take a second-order Taylor approximation of (64).

$$\tilde{\Delta}_{w,t}=\frac{\partial\Delta_{w,t}}{\partial\pi_w}\tilde{\pi}_{w,t}+\frac{\partial\Delta_{w,t}}{\partial\pi}\tilde{\pi}_{t-1}+\frac{\partial\Delta_{w,t}}{\partial\Delta_w}\tilde{\Delta}_{w,t-1}+\frac{1}{2}\frac{\partial^2\Delta_{w,t}}{\partial\pi_w^2}\tilde{\pi}_{w,t}^2+\frac{1}{2}\frac{\partial^2\Delta_{w,t}}{\partial\pi^2}\tilde{\pi}_{t-1}^2+$$
$$+\frac{1}{2}\frac{\partial^2\Delta_{w,t}}{\partial\Delta_w^2}\tilde{\Delta}_{w,t-1}^2+\frac{\partial^2\Delta_{w,t}}{\partial\pi_w\partial\pi}\tilde{\pi}_{w,t}\tilde{\pi}_{t-1}+\frac{\partial\Delta_{w,t}}{\partial\pi_w\partial\Delta_w}\tilde{\pi}_{w,t}\tilde{\Delta}_{w,t-1}+\frac{\partial\Delta_{w,t}}{\partial\pi\partial\Delta_w}\tilde{\pi}_{t-1}\tilde{\Delta}_{w,t-1}+$$
$$\mathcal{O}\left(\|\zeta\|^3\right)$$

$\frac{\partial\Delta_{w,t}}{\partial\pi_w}=0$

$\frac{\partial\Delta_{w,t}}{\partial\pi}=0$

$\frac{\partial\Delta_{w,t}}{\partial\Delta_w}=\xi_w$

$\frac{\partial^2\Delta_{w,t}}{\partial\pi_w^2}=\varphi\left(1+\sigma_L\right)\left(\varphi\sigma_L+1\right)\frac{\xi_w}{1-\xi_w}$

$\frac{\partial^2\Delta_{w,t}}{\partial\pi^2}=\gamma_w^2\varphi\left(1+\sigma_L\right)\left(\varphi\sigma_L+1\right)\frac{\xi_w}{1-\xi_w}$

$\frac{\partial^2\Delta_{w,t}}{\partial\Delta_w^2}=0$

$$\frac{\partial^2 \Delta_{w,t}}{\partial \pi_w \partial \pi} = -\gamma_w \varphi (1+\sigma_L)(\varphi \sigma_L + 1) \frac{\xi_w}{1-\xi_w}$$
$$\frac{\partial^2 \Delta_{w,t}}{\partial \pi_w \partial \Delta_w} = \xi_w \varphi (1+\sigma_L)$$
$$\frac{\partial \Delta_{w,t}}{\partial \pi \partial \Delta_w} = -\xi_w \gamma_w \varphi (1+\sigma_L)$$

Using the derivatives and replacing the variables with a til for the deviations from the steady state in logs, we reach

$$\hat{\Delta}_{w,t} = \xi_w \hat{\Delta}_{w,t-1} + \frac{1}{2} \varphi (1+\sigma_L)(\varphi \sigma_L + 1) \frac{\xi_w}{1-\xi_w} \left(\hat{\pi}_{w,t}^2 + \gamma_w^2 \hat{\pi}_{t-1}^2 - 2\gamma_w \hat{\pi}_{w,t} \hat{\pi}_{t-1} \right) + \mathcal{O}\left(\|\zeta\|^3 \right)$$

This result takes note of the fact that $\hat{\Delta}_{w,t}$ is already a term of second-order.

D.2.2 Approximation of aggregate labour and price dispersion measure

Consider the variable $\Delta_{p,t} = \int_0^n \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di$ as the price dispersion measure. Recall equation (10):

$$L_t = \left[\left(\frac{1}{n} \right)^{\frac{1}{\varphi}} \int_0^n l_t(i)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}}$$

By equation (18), we can define the individual labour and replace in the expression for the aggregate labour:

$$Y_t(i) = A_t l_t(i) - \Phi \Leftrightarrow l_t(i) = \frac{Y_t(i) + \Phi}{A_t}$$
$$L_t = \left[\left(\frac{1}{n} \right)^{\frac{1}{\varphi}} \int_0^n \left(\frac{Y_t(i) + \Phi}{A_t} \right)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}} \Leftrightarrow$$
$$L_t = \left[\left(\frac{1}{n} \right)^{\frac{1}{\varphi}} \int_0^n \left(\frac{\left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t}{n A_t} + \frac{\Phi}{A_t} \right)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}}$$

by equation (19). By Hölder's inequality[20], we get

[20] Hölder's inequality states that $\int_a^b f(x) g(x) dx \leq \left(\int_a^b f(x)^p dx \right)^{\frac{1}{p}} \left(\int_a^b g(x)^q dx \right)^{\frac{1}{q}}$ when $\frac{1}{p} + \frac{1}{q} = 1$. In our case, we have $f(x) = \left(\frac{\left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t}{n A_t} + \frac{\Phi}{A_t} \right)^{\frac{\varphi-1}{\varphi}}$, $g(x) = 1$, $p = \frac{\varphi}{\varphi-1}$ and therefore $q = \varphi$.

$$\left[\int_0^n \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t}{nA_t} + \frac{\Phi}{A_t}\right)^{\frac{\varphi-1}{\varphi}} di\right]^{\frac{\varphi}{\varphi-1}} \leq \left\{\left[\int_0^n \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t}{nA_t} + \frac{\Phi}{A_t}\right) di\right]^{\frac{\varphi-1}{\varphi}} \left(\int_0^n 1di\right)^{\frac{1}{\varphi}}\right\}^{\frac{\varphi}{\varphi-1}} =$$

$$= \int_0^n \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t}{nA_t} + \frac{\Phi}{A_t}\right) di \times n^{\frac{1}{\varphi-1}}$$

Consequently,

$$L_t \leq \left(\frac{1}{n}\right)^{\frac{1}{\varphi-1}} n^{\frac{1}{\varphi-1}} \int_0^n \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t}{nA_t} + \frac{\Phi}{A_t}\right) di \Leftrightarrow$$

$$L_t \leq \frac{Y_t}{A_t} \int_0^n \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di + \frac{n\Phi}{A_t} \Leftrightarrow$$

$$L_t = c\left(\frac{Y_t}{A_t}\Delta_{p,t} + \frac{n\Phi}{A_t}\right)$$

where $0 < c \leq 1$ is a unknown constant that guarantees the equality.

Now take a second-order Taylor expansion of L_t:

$$\tilde{L}_t \approx \frac{\partial L_t}{\partial Y}\tilde{Y}_t + \frac{\partial L_t}{\partial A}\tilde{A}_t + \frac{\partial L_t}{\partial \Delta_p}\tilde{\Delta}_{p,t} + \frac{1}{2}\frac{\partial^2 L_t}{\partial Y^2}\tilde{Y}_t^2 + \frac{1}{2}\frac{\partial^2 L_t}{\partial A^2}\tilde{A}_t^2 + \frac{1}{2}\frac{\partial^2 L_t}{\partial \Delta_p^2}\tilde{\Delta}_{p,t}^2 +$$
$$+ \frac{\partial^2 L_t}{\partial Y \partial A}\tilde{Y}_t\tilde{A}_t + \frac{\partial^2 L_t}{\partial Y \partial \Delta_p}\tilde{Y}_t\tilde{\Delta}_{p,t} + \frac{\partial^2 L_t}{\partial A \partial \Delta_p}\tilde{A}_t\tilde{\Delta}_{p,t} + \mathcal{O}\left(\|\zeta\|^3\right)$$

$$\frac{\partial L_t}{\partial Y} = \frac{c}{A}\Delta_p = \frac{c}{A}$$

since $\Delta_p = 1$, which implies that $L = c\left(\frac{Y + n\Phi}{A}\right)$.

$$\frac{\partial L_t}{\partial A} = c\left(-\frac{Y\Delta_p}{A^2} - \frac{n\Phi}{A^2}\right) = -c\left(\frac{Y + n\Phi}{A^2}\right)$$

$$\frac{\partial L_t}{\partial \Delta_p} = \frac{cY}{A}$$

$$\frac{\partial^2 L_t}{\partial Y^2} = 0$$

$$\frac{\partial^2 L_t}{\partial A^2} = 2c\left(\frac{Y + n\Phi}{A^3}\right)$$

$$\frac{\partial^2 L_t}{\partial \Delta_p^2} = 0$$

$$\frac{\partial^2 L_t}{\partial Y \partial A} = -\frac{c}{A^2}$$

$$\frac{\partial^2 L_t}{\partial Y \partial \Delta_p} = \frac{c}{A}$$

$$\frac{\partial^2 L_t}{\partial A \partial \Delta_p} = -\frac{cY}{A^2}$$

Replacing the calculated derivatives in the above expression for $\tilde{L}_t$:

$$L\left(\hat{L}_t + \frac{1}{2}\hat{L}_t^2\right) = \frac{cY}{A}(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2) - c\left(\frac{Y + n\Phi}{A}\right)(\hat{A}_t + \frac{1}{2}\hat{A}_t^2) + \frac{cY}{A}\hat{\Delta}_{p,t} +$$

$$+c\left(\frac{Y+n\Phi}{A}\right)\hat{A}_t{}^2-\frac{cY}{A}\hat{Y}_t\hat{A}_t+\mathcal{O}\left(\|\zeta\|^3\right)\Leftrightarrow$$

$$L\left(\hat{L}_t+\frac{1}{2}\hat{L}_t^2\right)=\frac{cY}{A}\left(\hat{Y}_t+\frac{1}{2}\hat{Y}_t^2+\hat{\Delta}_{p,t}-\hat{Y}_t\hat{A}_t\right)-c\left(\frac{Y+n\Phi}{A}\right)\left(\hat{A}_t+\frac{1}{2}\hat{A}_t^2-\hat{A}_t{}^2\right)+$$
$$\mathcal{O}\left(\|\zeta\|^3\right)\Leftrightarrow$$

$$\hat{L}_t+\frac{1}{2}\hat{L}_t^2=\frac{Y}{Y+n\Phi}\left[\hat{Y}_t\left(1+\frac{1}{2}\hat{Y}_t-\hat{A}_t\right)+\hat{\Delta}_{p,t}\right]-\left(\hat{A}_t-\frac{1}{2}\hat{A}_t^2\right)+\mathcal{O}\left(\|\zeta\|^3\right)\Leftrightarrow$$

$$\hat{L}_t+\frac{1}{2}\hat{L}_t^2=(1+n\phi)^{-1}\left[\hat{Y}_t\left(1+\frac{1}{2}\hat{Y}_t-\hat{A}_t\right)+\hat{\Delta}_{p,t}\right]-\left(\hat{A}_t-\frac{1}{2}\hat{A}_t^2\right)+\mathcal{O}\left(\|\zeta\|^3\right)\Leftrightarrow$$

$$\hat{L}_t=(1+n\phi)^{-1}\left[\hat{Y}_t\left(1+\frac{1}{2}\hat{Y}_t-\hat{A}_t\right)+\hat{\Delta}_{p,t}\right]-\left(\hat{A}_t-\frac{1}{2}\hat{A}_t^2\right)-\frac{1}{2}\hat{L}_t^2+\mathcal{O}\left(\|\zeta\|^3\right)\quad(65)$$

This result takes into account that $\hat{\Delta}_{p,t}$ is already a term of second-order (recall that $\phi=\frac{\Phi}{Y}$).

The law of motion of $\Delta_{p,t}$ will be similar to what was determined for $\Delta_{w,t}$. First, expand $\Delta_{p,t}$ as a sum (where P_t^* is the optimal wage at date t)

$$\Delta_{p,t}=\left(1-\xi_p\right)\left(\frac{P_t^*}{P_t}\right)^{-\theta}+\xi_p\left(1-\xi_p\right)\left(\frac{P_{t-1}^*\pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\theta}+$$
$$+\xi_p^2\left(1-\xi_p\right)\left(\frac{P_{t-2}^*\pi_{t-2}^{\gamma_p}\pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\theta}+\xi_p^3\left(1-\xi_p\right)\left(\frac{P_{t-3}^*\pi_{t-3}^{\gamma_p}\pi_{t-2}^{\gamma_p}\pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\theta}+\ldots\Leftrightarrow$$

$$\Delta_{p,t}=\left(1-\xi_p\right)\left(\frac{P_t^*}{P_t}\right)^{-\theta}+\sum_{j=1}^{\infty}\xi_p^j\left(1-\xi_p\right)\left(\frac{P_{t-j}^*\prod_{i=1}^{j}\pi_{t-i}^{\gamma_p}}{P_t}\right)^{-\theta}\quad(66)$$

One period earlier we have:

$$\Delta_{p,t-1}=\left(1-\xi_p\right)\left(\frac{P_{t-1}^*}{P_{t-1}}\right)^{-\theta}+\sum_{j=1}^{\infty}\xi_p^j\left(1-\xi_p\right)\left(\frac{P_{t-1-j}^*\prod_{i=1}^{j}\pi_{t-1-i}^{\gamma_p}}{P_{t-1}}\right)^{-\theta}\Leftrightarrow$$

$$\xi_p\pi_t^{\theta}\pi_{t-1}^{-\gamma_p\theta}\Delta_{p,t-1}=\sum_{j=1}^{\infty}\xi_p^j\left(1-\xi_p\right)\left(\frac{P_{t-j}^*\prod_{i=1}^{j}\pi_{t-i}^{\gamma_p}}{P_t}\right)^{-\theta}$$

Replacing for the second term in the right-hand side of equation (66):

$$\Delta_{p,t}=\left(1-\xi_p\right)\left(\frac{P_t^*}{P_t}\right)^{-\theta}+\xi_p\pi_t^{\theta}\pi_{t-1}^{-\gamma_p\theta}\Delta_{p,t-1}\quad(67)$$

From equation (24) we get:

$$P_t^{1-\theta} = \xi_p \left(P_{t-1}\pi_{t-1}^{\gamma_p}\right)^{1-\theta} + \left(1-\xi_p\right)\left(P_t^*\right)^{1-\theta} \Leftrightarrow$$
$$1 = \xi_p \left(\frac{P_{t-1}}{P_t}\pi_{t-1}^{\gamma_p}\right)^{1-\theta} + \left(1-\xi_p\right)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} \Leftrightarrow$$
$$\frac{P_t^*}{P_t} = \left(\frac{1-\xi_p\pi_t^{\theta-1}\pi_{t-1}^{\gamma_p(1-\theta)}}{1-\xi_p}\right)^{\frac{1}{1-\theta}}$$

Replacing in equation (67):

$$\Delta_{p,t} = \xi_p\pi_t^{\theta}\pi_{t-1}^{-\gamma_p\theta}\Delta_{p,t-1} + \left(1-\xi_p\right)\left(\frac{1-\xi_p\pi_t^{\theta-1}\pi_{t-1}^{\gamma_p(1-\theta)}}{1-\xi_p}\right)^{\frac{-\theta}{1-\theta}} \tag{68}$$

Now take a second-order Taylor approximation of (68):

$$\tilde{\Delta}_{p,t} = \frac{\partial\Delta_{p,t}}{\partial\pi_t|\pi}\tilde{\pi}_t + \frac{\partial\Delta_{p,t}}{\partial\pi_{t-1}|\pi}\tilde{\pi}_{t-1} + \frac{\partial\Delta_{p,t}}{\partial\Delta_p}\tilde{\Delta}_{p,t-1} + \frac{1}{2}\frac{\partial^2\Delta_{p,t}}{\partial\pi_t|\pi^2}\tilde{\pi}_t^2 + \frac{1}{2}\frac{\partial^2\Delta_{p,t}}{\partial\pi_{t-1}|\pi^2}\tilde{\pi}_{t-1}^2 +$$
$$+\frac{1}{2}\frac{\partial^2\Delta_{p,t}}{\partial\Delta_p^2}\tilde{\Delta}_{p,t-1}^2 + \frac{\partial^2\Delta_{p,t}}{\partial\pi_t\partial\pi_{t-1}|\pi}\tilde{\pi}_t\tilde{\pi}_{t-1} + \frac{\partial^2\Delta_{p,t}}{\partial\pi_t\partial\Delta_p|\pi}\tilde{\pi}_t\tilde{\Delta}_{p,t-1} + \frac{\partial^2\Delta_{p,t}}{\partial\pi_{t-1}\partial\Delta_p|\pi}\tilde{\pi}_{t-1}\tilde{\Delta}_{p,t-1} +$$
$$\mathcal{O}\left(\|\zeta\|^3\right)$$

$$\frac{\partial\Delta_{p,t}}{\partial\pi_t|\pi} = 0$$
$$\frac{\partial\Delta_{p,t}}{\partial\pi_{t-1}|\pi} = 0$$
$$\frac{\partial\Delta_{p,t}}{\partial\Delta_p} = \xi_p$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\pi_t|\pi^2} = \frac{\xi_p\theta}{1-\xi_p}$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\pi_{t-1}|\pi^2} = \frac{\theta\xi_p\gamma_p^2}{1-\xi_p}$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\Delta_p^2} = 0$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\pi_t\partial\pi_{t-1}|\pi} = -\frac{\xi_p\theta\gamma_p}{1-\xi_p}$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\pi_t\partial\Delta_p|\pi} = \xi_p\theta$$
$$\frac{\partial^2\Delta_{p,t}}{\partial\pi_{t-1}\partial\Delta_p|\pi} = -\gamma_p\theta\xi_p$$

Then,

$$\hat{\Delta}_{p,t} + \frac{1}{2}\hat{\Delta}_{p,t}^2 = \xi_p\left(\hat{\Delta}_{p,t-1} + \frac{1}{2}\hat{\Delta}_{p,t-1}^2\right) + \frac{1}{2}\frac{\xi_p\theta}{1-\xi_p}\left(\hat{\pi}_t + \frac{1}{2}\hat{\pi}_t^2\right)^2 + \frac{1}{2}\frac{\theta\xi_p\gamma_p^2}{1-\xi_p}\left(\hat{\pi}_{t-1} + \frac{1}{2}\hat{\pi}_{t-1}^2\right)^2 -$$
$$-\frac{\xi_p\theta\gamma_p}{1-\xi_p}\left(\hat{\pi}_t + \frac{1}{2}\hat{\pi}_t^2\right)\left(\hat{\pi}_{t-1} + \frac{1}{2}\hat{\pi}_{t-1}^2\right) + \xi_p\theta\left(\hat{\pi}_t + \frac{1}{2}\hat{\pi}_t^2\right)\left(\hat{\Delta}_{p,t-1} + \frac{1}{2}\hat{\Delta}_{p,t-1}^2\right) -$$
$$-\gamma_p\theta\xi_p\left(\hat{\pi}_{t-1} + \frac{1}{2}\hat{\pi}_{t-1}^2\right)\left(\hat{\Delta}_{p,t-1} + \frac{1}{2}\hat{\Delta}_{p,t-1}^2\right) + \mathcal{O}\left(\|\zeta\|^3\right) \Leftrightarrow$$
$$\hat{\Delta}_{p,t} = \xi_p\hat{\Delta}_{p,t-1} + \frac{1}{2}\frac{\xi_p\theta}{1-\xi_p}\left(\hat{\pi}_t^2 + \gamma_p^2\hat{\pi}_{t-1}^2 - 2\gamma_p\hat{\pi}_t\hat{\pi}_{t-1}\right) + \mathcal{O}\left(\|\zeta\|^3\right)$$

This result takes note of the fact that $\hat{\Delta}_{p,t}$ is already a term of second-order.

Recall the expression of the disutility from working (61):

$$V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t}=\bar{V}\left(1+\hat{\Delta}_{w,t}\right)+\bar{V}_L L\left[\hat{L}_t+\frac{(1+\sigma_L)\hat{L}_t^2}{2}+\hat{L}_t\left(\hat{\varepsilon}_t^b+\hat{\varepsilon}_t^L\right)\right]+\mathcal{O}\left(\|\zeta\|^3\right)$$

We can rewrite this expression by taking into account that $\bar{V}=\dfrac{\bar{V}_L L}{1+\sigma_L}$:

$$V(L_t,\varepsilon_t^b,\varepsilon_t^L)\Delta_{w,t}=\bar{V}_L L\left(\frac{1+\hat{\Delta}_{w,t}}{1+\sigma_L}+\hat{L}_t+\frac{(1+\sigma_L)\hat{L}_t^2}{2}+\hat{L}_t\left(\hat{\varepsilon}_t^b+\hat{\varepsilon}_t^L\right)\right)+\mathcal{O}\left(\|\zeta\|^3\right)$$

D.3 Welfare expression

Now take the present discounted sum of the welfare equation (60):

$$\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t=\sum_{t=0}^{\infty}\beta^t\left[U(C_t,H_t,\varepsilon_t^b)-\frac{1}{n}\int_0^n V(L_t(i),\varepsilon_t^b,\varepsilon_t^L)di\right]=$$

$$=\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}\bar{U}+\bar{U}_C C\left[\begin{array}{c}\left(\hat{C}_t-h\hat{C}_{t-1}\right)+\frac{1}{2}\left(\hat{C}_t^2-h^2\hat{C}_{t-1}^2\right)+\dfrac{1-h}{2(1-\sigma_c)}\hat{\varepsilon}_t^{b\,2}-\\-\dfrac{\sigma_c}{2(1-h)}\left(\hat{C}_t-h^2\hat{C}_{t-1}\right)^2+\hat{\varepsilon}_t^b\left(\hat{C}_t-h^2\hat{C}_{t-1}\right)\end{array}\right]-\\-\bar{V}_L L\left(\dfrac{1+\hat{\Delta}_{w,t}}{1+\sigma_L}+\hat{L}_t+\dfrac{(1+\sigma_L)\hat{L}_t^2}{2}+\hat{L}_t\left(\hat{\varepsilon}_t^b+\hat{\varepsilon}_t^L\right)\right)\end{array}\right\}+$$

$$\mathcal{O}\left(\|\zeta\|^3\right)$$

Following Benigno and Woodford (2004), $\Theta=1-\dfrac{\theta-1}{\theta}\dfrac{\varphi-1}{\varphi}<1$ measures the inefficiency of the steady-state output level $\bar{Y}$, so that we can use the steady-state relation $\bar{V}_L L=(1-\Theta)\bar{U}_C C$.

Take into account that equations (64) and (68) can be integrated in order to have:

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{w,t}=\frac{1}{2}\varphi(1+\sigma_L)(\varphi\sigma_L+1)\frac{\xi_w}{(1-\xi_w)(1-\beta\xi_w)}\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_{w,t}^2+\gamma_w^2\hat{\pi}_{t-1}^2-2\gamma_w\hat{\pi}_{w,t}\hat{\pi}_{t-1}\right)+$$
$$\mathcal{O}\left(\|\zeta\|^3\right)$$

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{p,t}=\frac{1}{2}\frac{\xi_p\theta}{(1-\xi_p)(1-\beta\xi_p)}\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^2+\gamma_p^2\hat{\pi}_{t-1}^2-2\gamma_p\hat{\pi}_t\hat{\pi}_{t-1}\right)+\mathcal{O}\left(\|\zeta\|^3\right)$$

Replacing this in the welfare expression and remembering the expression for labour (65), we get, after some algebra:

$$\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t=$$

$$=\sum_{t=0}^{\infty}\beta^t\left\{\bar{U}+\bar{U}_C C\left[\begin{array}{c}\left(\hat{C}_t-h\hat{C}_{t-1}\right)+\frac{1}{2}\left(\hat{C}_t^2-h^2\hat{C}_{t-1}^2\right)-\dfrac{\sigma_c}{2(1-h)}\left(\hat{C}_t-h^2\hat{C}_{t-1}\right)^2+\\+\dfrac{1-h}{2(1-\sigma_c)}\hat{\varepsilon}_t^{b\,2}+\hat{\varepsilon}_t^b\left(\hat{C}_t-h^2\hat{C}_{t-1}\right)-u_1\left(\hat{\pi}_{w,t}^2+\gamma_w^2\hat{\pi}_{t-1}^2-2\gamma_w\hat{\pi}_{w,t}\hat{\pi}_{t-1}\right)-\\-u_2\left(\hat{\pi}_t^2+\gamma_p^2\hat{\pi}_{t-1}^2-2\gamma_p\hat{\pi}_t\hat{\pi}_{t-1}\right)-u_3\hat{Y}_t^2+u_4\hat{Y}_t\hat{A}_t-u_5\hat{Y}_t\left(1+\hat{\varepsilon}_t^b+\hat{\varepsilon}_t^L\right)\end{array}\right]\right\}+$$
$$+\mathcal{O}\left(\|\zeta\|^3\right)$$

$$u_1 = \frac{\xi_w \varphi (\varphi \sigma_L + 1)(1-\Theta)}{2(1-\xi_w)(1-\beta\xi_w)} > 0$$

$$u_2 = \frac{\xi_p \theta (1-\Theta)}{2(1-\xi_p)(1-\beta\xi_p)(1+n\phi)} > 0$$

$$u_3 = \frac{(1-\Theta)(1+n\phi+\sigma_L)}{2(1+n\phi)^2} > 0$$

$$u_4 = \frac{(1-\Theta)(1+n\phi-\sigma_L)}{(1+n\phi)^2} > 0 \text{ if } 1+n\phi > \sigma_L$$

$$u_5 = \frac{1-\Theta}{1+n\phi} > 0$$ [21]

The welfare expression at moment t used in section 4.2.3 is the following

$$\mathcal{W}_t = \bar{U} + \bar{U}_C C \left[\begin{array}{c} \left(\hat{C}_t - h\hat{C}_{t-1}\right) + \frac{1}{2}\left(\hat{C}_t^2 - h^2\hat{C}_{t-1}^2\right) - \frac{\sigma_c}{2(1-h)}\left(\hat{C}_t - h^2\hat{C}_{t-1}\right)^2 + \\ + \frac{1-h}{2(1-\sigma_c)}\,\hat{\varepsilon}_t^{b\,2} + \hat{\varepsilon}_t^b\left(\hat{C}_t - h^2\hat{C}_{t-1}\right) - u_1\left(\hat{\pi}_{w,t}^2 + \gamma_w^2\hat{\pi}_{t-1}^2 - 2\gamma_w\hat{\pi}_{w,t}\hat{\pi}_{t-1}\right) - \\ -u_2\left(\hat{\pi}_t^2 + \gamma_p^2\hat{\pi}_{t-1}^2 - 2\gamma_p\hat{\pi}_t\hat{\pi}_{t-1}\right) - u_3\hat{Y}_t^2 + u_4\hat{Y}_t\hat{A}_t - u_5\hat{Y}_t\left(1 + \hat{\varepsilon}_t^b + \hat{\varepsilon}_t^L\right) \end{array} \right]$$

[21] Obviously, for the foreign economy we need to replace n for $(1-n)$.

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